KAPAO: Design and Assembly of the Wavefront Sensor for an Adaptive Optics Instrument

by

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A thesis submitted in partial fulfillment for the degree of

Bachelor of Arts

in

Physics and Astronomy

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Pomona College

May 2013
KAPAO is a low-cost, dual-band (optical/IR), natural guide star adaptive optics (AO) instrument currently in development for the Pomona College 1-meter telescope at Table Mountain. The instrument is designed around a 140-actuator MEMS (microelectromechanical) deformable mirror and a Shack-Hartman wavefront sensor (WFS) using a high speed 80x80 pixel E2V CCD39. Here, we present the design and assembly of the wavefront sensing component of the AO instrument. The telescope pupil plane is broken into an array of 11x11 subapertures by a lenslet array and these subapertures are focused onto the CCD detector array. Each subaperture uses a 2x2 binned pixel array as a bicell detector. The mapping from the telescope to the DM actuators to the actual CCD subapertures is discussed in detail. This thesis also discusses the possibility of implementing adaptive wavefront sensing techniques. The current KAPAO WFS leg is designed for average seeing conditions with $r_0$ of 9.36 cm. The possibility of implementing lenslets with different focal lengths, which correspond to different fields of view, is discussed. There is also an investigation into implementing a WFS leg with larger subapertures, doubling the current subaperture size.
I would like to thank my advisor, Professor Philip Choi, for his guidance on this project. It has been a great pleasure to work under Professor Choi. I am incredibly grateful for his endless enthusiasm and expert advising over the past four years. I would also like to thank Professor Scott Severson of Sonoma State University for sharing his expertise in astronomical instrumentation. Much of the progress I have made on the testbed would have been impossible without his assistance. The KAPAO Team has also been a fantastic group to work with. Past members who have made amazing contributions include Alex Rudy, Blaine Gilbreth, and Will Morrison. Lorcan McGonigle has developed the models on which the entire KAPAO system is built. Erik Littleton has created an impressive software suite used for characterization of system performance. Jonathan Wong has devoted countless hours to in-lab development and characterization. The Robo-AO team of CalTech, including Christoph Baranec and Reed Riddle, have also been a huge help on the project during my time at Pomona. I would also like to thank Emily Chang for editing this document. Glenn Flohr and Tony Grigsby have also provided invaluable components that house many of the components currently on KAPAO.

Finally, I would like to thank my family. Para mi familia, gracias por tu apoyo a lo largo de los años. Tu valor me da fuerza.
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Chapter 1

Introduction

The distortion in astronomical images caused by the earth’s atmosphere are serious issues for performing science on ground based telescopes. The atmosphere can be modeled as layers of air with varying wind speeds and temperature pockets causing turbulence. This constant motion aberrates the light from a distant star within the last fraction of a second of its path to a telescope. Even on a night when the sky is clear and calm to the naked eye, ground based telescopes still have to contend with the limits that the disturbances in the air put on astronomical seeing.

Various strategies have been taken in order to minimize the effects of the atmosphere when imaging through a telescope. Observatories have been constructed on high peaks in order to minimize the amount of the atmosphere there is to look through. There is also the recent practice of launching telescopes into space to image above the atmosphere. However, imaging on high mountain tops can not entirely escape the effects of the atmosphere and sending telescopes into space is not financially practical for most institutions. Another practice, developed in the middle of the 20th century, is adaptive optics.

Adaptive optics (AO) is a technology developed to compensate for astronomical images that have been distorted by the atmosphere. Imaging systems with AO capabilities are able to take the distorted image and correct for the aberrations in the wavefront using wavefront sensing techniques.
1.1 History

Modern astronomical adaptive optics was first proposed in 1953 by Horace Babcock (Tyson 2011). Throughout the 1950’s scientists implemented simple telescope systems to correct for low-order aberrations known as tip and tilt, named for the angle of incidence of the wavefront. The science progressed slowly because the correction of many higher-order aberrations was impossible due to the lack of fast precision technology at the time (Hardy 1998).

Military applications, such as missile defense systems and laser technology, led to many advances in AO technology, although most of these advances were classified. One major key step in the development of AO, credited to William Happer, was the idea of using the layer of sodium in the earth’s atmosphere to create an artificial guide star with a laser. By imaging this artificial star, it is possible to measure the aberrations in the atmosphere and correct a region of the sky based on these aberrations. A majority of the work was classified until 1991. After the declassification of the work, the astronomy community was able to make use of these advances in AO. The developments in computing technology around the 1990’s also led to a period of improvement in real-time control and correction over the next 20 years. The technique soon became a practical application to use on large telescopes.
1.2 The Atmosphere

Layers of the atmosphere can be thought of as a series of lenses, each one contributing some small amount of aberration to the wavefront from a star. Temperature variations between layers in the atmosphere are on the order of about $< 1^\circ$C. These small temperature differences lead to turbulent motion, resulting in changes in wind speed and density. This causes changes in the index of refraction, $n$, from layer to layer. These changes are small, $\Delta n$ being on the order of $10^{-6}$. However, as a beam of light passes through more and more layers, these variations will accumulate leading to heavy aberration of the wavefront (Tyson 2011).

For a star, these compounding aberrations from layer to layer compromise the image. The uniformity of the planar wavefront is lost due to different sections of the wavefront passing through slightly different paths. This will cause distortion when the wavefront is imaged on a detector, such as scattering effects. The position of the star will also change slightly due to deflection of the entire wavefront. This can cause a smearing effect on the camera detector that occurs as one continually images, leading to problems for long exposure observations.

For larger telescopes, this can become a serious problem as correction becomes more difficult over a larger area. Along with capturing more photons from the sky, a larger aperture also has a larger area of the atmosphere to see through. This can cause optical path differences among the incident photons on opposite sides of the wavefront on the order of microns.

AO seeks to correct for atmospheric aberration, not by directly dealing with the atmosphere, but by normalizing the path of the incident photons that compose the wavefront. By undoing the aberration that the atmosphere has caused, AO restores the aberrated wavefront to a planar wave in order to resolve the blurred image to a diffraction limited point source.
1.3 A Short Description of AO

An adaptive optics system has three basic components. These are the wavefront sensor (WFS), which detects deformations in the wavefront of the light coming through the imaging system, an actuated deformable mirror (DM), which contorts to counter the effects on the wavefront caused by the atmosphere, and a control computer, which is able to handle the high speed mathematics and matrix multiplication and communicate between the WFS and DM.

In Figure 1.1, there is a plane wave entering the system. This may be a point source coming from infinity, such as a star. Before encountering the optics, the wavefront passes through a wave-distorting medium, which in our case is the atmosphere. Now, the aberrated wave will move through the optical system, and before it reaches the imaging device, a portion of the beam is redirected to the WFS. The function of the WFS is to detect the shape of the wavefront. There are a variety of wavefront sensing techniques that will be discussed further in Chapter 2.

This WFS data is relayed to a control computer that reconstructs the wavefront by computing...
a correction factor using a reconstructor matrix. This data is then sent to the DM which will adjust its shape to undo what the wave-distorting medium has done. The DM is usually an array of small microelectromechanical (MEMS) actuators that move independently of one another. The actuators are covered by a reflective surface, which is free to bend within the limits of the actuators. These limits are usually on the order of a couple of microns.

This all happens within a closed-loop about 1000 times a second in order for the system to keep up with the constantly changing atmosphere. The image will be corrected as the system enters closed loop stability. If the system is effective, the spot will approach the diffraction limit of the optical system.

1.4 KAPAO: Pomona Adaptive Optics

KAPAO is a National Science Foundation (NSF) funded project proposed to develop an adaptive optics system for the TMO 1-meter telescope (Figure 1.2). The project is a collaboration between Sonoma State University, Harvey Mudd College, and the Robo-AO group at the California Institute of Technology. A major goal for this project is to develop a community of cost-effective AO systems for small telescopes.

Actual construction of the system began in 2009 with the development of a testbed system and the optical design of the prototype model. The purpose of the testbed is to characterize each of the hardware components and the software in a lab environment. The prototype design, christened KAPAO Alpha, is also operational. It is based on cost-effective off-the-shelf optics and is able to take on-sky data. Alpha is also mobile between the telescope and the lab, in order to take data at TMO and further characterize the system in a lab environment.

Construction of Alpha began in the summer of 2010 and was completed the following spring. Since then, Alpha has been on the telescope numerous times. Currently a goal of the project is
1.4 KAPAO: Pomona Adaptive Optics

Figure 1.2 KAPAO Alpha, the prototype system attached onto the back of Pomona College’s TMO 1-meter telescope. *Credit: Will Morrison*

analysis of this data and quantification of on-sky closed-loop performance. Also during this time, the model of the final system was completed. KAPAO Prime, Alpha’s successor, was informed by the work performed on Alpha. The design is based on fully customized optics and it is the final system that will remain fixed on the telescope upon completion and characterization. In the future, KAPAO’s prototype systems will serve as testbeds on Pomona College’s campus, allowing undergraduate students the opportunity to work on a project that gives a multidisciplinary experience in STEM fields.

Throughout its history, KAPAO has been built upon the same three major components. The DM is a 12x12 actuator microelectromechanical systems (MEMS) device from Boston Micromachines.
The WFS is a Shack-Hartmann wavefront sensor using a SciMeasure 80x80 pixel E2V CCD39 camera. Thirdly a Linux computer runs the software and controls each of the components.

1.5 Purpose and Structure

This thesis is devoted to the construction and optimization of KAPAO Prime’s WFS leg. The focus will be on the development of an ideal prescription for the current WFS optical leg and possible modifications to the wavefront sensor that could benefit the system in the future.

Chapter 2 sets up the basic theory behind telescope optics and WFS techniques, devoting special attention to the Shack-Hartmann WFS, the technique that KAPAO uses. Chapter 3 discusses the implementation of a Shack-Hartmann WFS into the KAPAO system and what changes are necessary to the basic design set up in Chapter 2. In Chapter 4 we are introduced to hardware implementations that improve the functionality of KAPAO over a broader range of conditions. Chapter 5 will delve into methods to improve the efficiency of WFS spot detection in software, as well as modifications that are necessary to the software for some of the hardware modifications in Chapter 4.

Careful attention will be paid to the science behind wavefront sensing as well as improvements to the software. A report on the reflectivities and transmittances of the optics will also be included as well as a user guide for the WFS. This work will also address the future of KAPAO after the completion and on-sky implementation of Prime to some detail.
Chapter 2

Theory

This section uses Tyson’s Principles of Adaptive Optics (Tyson 2011) and Hardy’s Adaptive Optics (Hardy 1998) as references.

An AO system is composed of many different parts, and talking in detail about each of these requires a solid understanding and long discussion of a different area of physics and engineering, from photonics to microelectromechanical devices. The focus of this chapter is mainly on the theory behind wavefront sensing and the various wavefront sensing techniques in use in modern AO systems. There will also be a short exposition on the basic optical theory involved.

Wavefront sensing is a technique used to either directly or indirectly measure the degree of aberration of a wavefront. Direct approaches use measurements of the phase of the wavefront and measure optical path differences across the wavefront, while indirect approaches use the phase information as a signal to perform corrections, rather than directly calculating the wavefront. This project uses direct wavefront sensing techniques.

To apply corrections in real-time, the system must sense quickly enough and with enough spatial resolution for corrections to be effective. This requires measurement of pulses on extremely short time scales, in some cases on the order of 2\(\mu\)s. High spatial resolution requires a high density of parallel sensing across the aperture diameter, enough such that the largest unit on the detector
has less than 1 radian of wavefront error from the atmosphere. This can usually be around 10 cm.

There are many sensing techniques, each with its own metric. The current KAPAO system uses a Shack-Hartmann wavefront sensor. Other methods include shearing interferometry, curvature sensing, and pyramid sensing, each of which this chapter will discuss, beginning with a short discussion of basic optics.

2.1 Basic Optical Theory of Telescopes

2.1.1 Single Source

Figure 2.1 A simplified two-lens system with an on-axis source. The source from infinity coming in as a collimated beam is focused and recollimated by another lens at the appropriate focal distance. The chief ray is a dashed line.

Being a point source, a star radiates isotropically. This produces a spherical distribution of light. As this distribution approaches earth from thousands of light-years away, we can assume that the light is coming from infinity and approximate the wavefront as planar. In Figure 2.1, a point source is coming from infinity and is brought to a focus by an objective lens of focal length
2.1 Basic Optical Theory of Telescopes

Before this light reaches the objective lens the light is collimated, meaning the rays of the light ideally do not diverge or converge over distance. This light is then recollimated by a second lens with a focal length of \( f_2 \) at this appropriate distance. This beam is also now magnified due to the ratio of the focal lengths.

This second lens could be an eyepiece, which recollimates the beam, this time magnified, in order to image the stars with the lens of your eye. Alternatively another lens may be used after this collimating beam to focus the point source and image it with a detector, such as a charge-coupled device (CCD). The magnification is given by the ratio of the size of the beam before and after encountering the two lens system. This ratio is also equal to the ratio of the focal lengths of the lenses used, if the distances are set appropriately. This ratio is,

\[
M = \frac{d_1}{d_2} = \frac{f_1}{f_2}
\]  

(2.1)

2.1.2 Multiple Sources

Being a telescope system, we will want to image multiple sources. In Figure 2.2 we see a source coming in along the optical axis (red) and a source coming in off-axis (green). These two separate stars are imaged at the focal plane. Placing a CCD at this point produces an image of two stars separated by some distance determined by the angle of incidence.

Imaging two separate sources coming into the objective at different angles relative to each other requires a discussion of the pupil plane. The pupil plane is the region of space within the optical system where the rays of the collimated light overlap. This occurs at the telescope objective, where the rays all overlap to enter the system and at recollimated points at the appropriate focal length of the recollimating lens, such as a distance \( f_2 \) after the second lens in Figure 2.2.

Within the system there are multiple pupil planes, and because all the rays in the system overlap
Figure 2.2 A simplified two-lens system with an on-axis source and an off-axis source. The chief rays are marked by a dashed line. The focal plane and the pupil planes are labeled.

at each of these points altering the wavefront at one pupil plane will have the same effect at every other pupil plane. This means that the planes are conjugate to each other, and therefore directly related to the original pupil plane, at the telescope objective. By placing correcting elements at a pupil plane with a chosen natural guide star as our point source, the system is able to correct for the aberrations in multiple point sources at varying angles at this overlapping pupil plane. The importance of the pupil plane in Shack-Hartmann wavefront sensing will be further discussed in Chapter 3.

2.2 Modal and Zonal Representation Methods

The wavefront can be expressed in two ways, zonal and modal. Zonal wavefront sensing will break up the aperture into smaller segments and find the shape of the wavefront from analysis of these segments. This is usually done by measuring a local slope of the wavefront at a segment, and nu-
2.3 Shack-Hartmann Wavefront Sensing

2.3.1 Description

The Shack-Hartmann wavefront sensor (SH-WFS) is a technique that has gained popularity in AO due to its simplicity. The method relies on an array of microlenses and a detector array, as shown in Figure 2.3.

The wavefront in collimated space at the pupil plane is incident on the microlens array. The array will then break up the wavefront, focusing spots onto the detector. Each of these spots of light is focused onto a subaperture. A perfectly flat wavefront will produce an array of spots where the spots land on the optical axis of their respective lenslet. Any deviation in the angle of incidence numerically integrating these slopes across the aperture. Modal wavefront sensing encompasses the entire aperture and expresses the wavefront in terms of coefficients of a polynomial expression. In most cases this polynomial expression is the set of Zernike polynomials across a circular aperture.

Most systems will work mainly in a zonal basis. This is because the zonal method relies on segmentation of the wavefront. This can be more easily expanded for higher spatial resolution on a system, by using more segmentation. For modal sensing, the lower order Zernike polynomials describe familiar aberrations, such as tip, tilt, and focus, and are easy to expand, but for higher-order terms calculations become increasingly difficult. For this same reason, in systems that employ both methods modal orders are more reliable and commonly used for lower order errors, while higher order errors will be improved using zonal compensation.

In either case, both methods will produce an adequate description of the wavefront. It is possible to convert the wavefront produced using the zonal method to get modal information from the wavefront and vice versa. The conversion is useful for data analysis of modal information in the wavefront for data processing.
2.3 Shack-Hartmann Wavefront Sensing

Figure 2.3 Diagram of a lenslet array focusing a single axis array of spots onto a detector for an ideal planar wavefront (left) and an aberrated wavefront (right). The dark blue circles denote where the spots should fall for an ideal planar wavefront and the red dots denote the focused spots on the detector.

of a portion of the wavefront onto a lenslet will cause a displacement from this central position.

2.3.2 The Bicell Detector

In order to detect this deviation in two dimensions on the CCD, we use a bicell detector as a sensor for our subaperture (Figure 2.4). This detector is an array of four pixels in a square centered on the optical axis, \( z \). Larger pixel arrays are possible; however the required processing power for larger arrays outweighs the gains in sensitivity. Displacements are measured in the \( x \) and \( y \) directions on the detector by the intensity offset of a deviated spot from a centered spot.

The calculation is performed by centroiding across the pixels. Assuming \( a, b, c, \) and \( d \) are the readouts from the CCD in each of the pixels in the bicell, and \( I \) is the summation of the 4 pixels, the displacement for the \( x \) and \( y \) axes are given by

\[
X = \frac{(a+d) - (b+c)}{I}
\]

(2.2)
2.3 Shack-Hartmann Wavefront Sensing

Figure 2.4 A single lenslet array focusing a spot onto a bicell detector. We also see the spot intensity pattern, labeling the effective width of the spot. (Hardy 1998)

\[ Y = \left( \frac{a+b}{f} \right) - \left( \frac{c+d}{f} \right) \]

(2.3)

It is important to consider the size of the spot in order to understand the range of the detector.

The centralized intensity distribution of a square subaperture, in our case the bicell detector, is given by

\[ I(x,y) = I_0 \left( \frac{\sin ax}{ax} \right)^2 \left( \frac{\sin ay}{ay} \right)^2 \]

(2.4)

where \( a = \frac{\pi d}{\lambda z} \), \( d \) is the subaperture size, and \( \lambda \) is the mean wavelength. As stated earlier, \( x \) and \( y \) are displacements in the plane of the detector. The first minimum of this intensity pattern occurs at \( x = y = \frac{\lambda z}{d} \), meaning that we get an effective central spot width of

\[ \Delta x = \frac{2\lambda z}{d} \]

(2.5)

2.3.3 Dynamic Range

Now we will discuss measuring the offset of a spot on the detector. For a single lenslet focusing a spot onto the detector, the range of the detector is limited by the width of the spot. Figure 2.5 shows the normalized output of a single axis of a bicell detector corresponding to the displacement.
of the spot, given in waves of tilt. Within the bounds of half a wavelength of tilt, the curve is approximately linear. Displacing the intensity pattern by a full wavelength will move the spot entirely into one half of the detector and any further motion will not change the response.

![Normalized bicell output](image)

Figure 2.5 Transfer function of a bicell detector along one axis with a single point source. Range is limited to within ±0.5 waves of tilt. (Hardy 1998)

This limit in the bicell’s dynamic range in wave tilt response is given in radians by

\[ \alpha_B = \frac{\lambda}{2d} \]  

(2.6)

Given this limitation in linear range on the detector, we now need to look at this coupled with limitations in atmospheric seeing. By looking at the atmospheric wavefront error over an aperture of size \( D \) in object space (on the telescope primary) and setting this root mean-square error equal to the linear range of the detector we get

\[ d' = 1.21r_0 \left( \frac{\lambda_{ref}}{\lambda} \right)^{6/5} \]  

(2.7)
\( d' \) is the size of a subaperture over which the RMS atmospheric wavefront tilt error is equal to the limit of linear range of our detector. \( r_0 \) is the Fried Parameter, a measure of atmospheric quality over a characteristic area on the telescope specified at a wavelength \( \lambda \), and \( \lambda_{ref} \) is the wavelength of our reference source (Hardy 1998). For TMO, the Fried Parameter is 9 cm.

We could improve the linear range by changing the size of the spot on the detector. Making the spots larger by defocusing will improve dynamic range, at the cost of sensitivity on the detector when centroiding. Another method would be to use a detector with more cells. One other common array is a 4x4 grid of detectors.

### 2.3.4 Spot Size and Focal Length

We can find the maximum lenslet focal length necessary for the spot to be entirely contained within one quadrant of the detector. If we go back to Equation 2.5, the assumed size of our spot, and equate this to the size of a quadrant of the bicell (width of \( d/2 \)), we get a maximum focal length (distance \( z \)) of

\[
z = \frac{d^2}{4\lambda}
\]

The minimum focal length is arbitrary and depends on the performance desired for specific seeing conditions.

### 2.3.5 Displacement Calculation Error

The random error in this measurement is a factor of spot size. It can be shown that the standard deviation of the angular position error along one axis due to random noise is given by

\[
\sigma_1 = \frac{3\pi \lambda}{16 \ d \ SNR}
\]
where SNR is the signal-to-noise ratio of the readout signal from the quadrant detector (Hardy 1998). This is true for a point source, when the wavefront is flat over the subaperture \((r_0 > d)\). When the reference source is large, such that \(\theta >> \lambda/d\), the position error is given by the source size

\[
\sigma_2 = \frac{\pi \theta}{8 \text{SNR}}
\]  
(2.10)

Combining these equations gives the expression for error in angular position on the detector

\[
\sigma_\alpha = \frac{\pi}{8 \text{SNR}} \left[ \left( \frac{3\lambda}{2d} \right)^2 + \theta^2 \right]^{1/2}
\]  
(2.11)

When there is turbulence across the subaperture \((r_0 < d)\) we can replace \(d\) in the equation with \(r_0\)

This measurement error underestimates the error on actual sensors. This is because the error in Equation 2.11 does not account for the separation between the detector segments. This slight error increase can be modeled with a constant \(K_g\). This constant is usually between 1.2 and 1.5 (Hardy 1998). It is also convenient to think of wavefront error as phase difference in radians. We can multiply by a factor of \(2\pi d/\lambda\). We end up with

\[
\sigma_\phi = \frac{\pi^2 K_g}{4(\text{SNR})} \left[ \left( \frac{3d}{2r_0} \right)^2 + \left( \frac{\theta d}{\lambda} \right)^2 \right]^{1/2}
\]  
(2.12)

\[
\sigma_\phi = \frac{\pi^2 K_g}{4(\text{SNR})} \left[ \left( \frac{3d}{2r_0} \right)^2 + \left( \frac{\theta d}{\lambda} \right)^2 \right]^{1/2}
\]  
(2.13)

### 2.4 Other Wavefront Sensing Techniques

This section discusses other wavefront sensing techniques available to adaptive optics systems.
2.4 Other Wavefront Sensing Techniques

2.4.1 Shearing Interferometry

A shearing interferometer is shown in Figure 2.6. In the method shown in figure 6, the incident wavefront is split into two beams by a wedged plate beam splitter, one reflected off the front edge and the other reflected off the back edge. These beams then interfere with each other, but without completely overlapping. The distance of the overlap is measured by $s$, the shearing distance. The intensity pattern within the overlap will contain phase information.

![Shearing interferometry setup](image)

**Figure 2.6** Shearing interferometry setup, depicting the fringe patterns in the overlap of the wavefronts *Credit: McGraw-Hill Science and Technology Dictionary*

For a plane wavefront $\phi'(x)$ interfering with a wavefront $\phi(x)$, the intensity pattern is given by the phase difference

$$I = \phi(x) - \phi'(x) = \phi(x)$$  \hspace{1cm} (2.14)

Now if we shear the wavefront, offsetting it by a distance $s$, the interference produces

$$I = \phi(x - s) - \phi(x)$$  \hspace{1cm} (2.15)
Normalizing the intensity by the shear distance $s$, we get

$$I = \frac{\phi(x-s) - \phi(x)}{s}$$

(2.16)

Now we can see that as $s$ approaches zero, this becomes the derivative of the wavefront, $d\phi/dx$ (Tyson 2011).

Shearing interferometer wavefront sensors are useful because the method is self-referencing. There is no reference plane wavefront necessary since the wavefront interferes with itself.

It is also possible to radially shear the wavefronts. This involves magnifying one of the beams to the point where we can assume that phase variations are small. Then the other beam is directed to the center of this effectively constant phase reference. The interference pattern will give information on the spherically symmetric aberrations in the wavefront. Rotational shearing is another method that involves a rotation of $180^\circ$.

Lateral shearing, the method described in figure 6, using a diffraction grating rather than a wedge is an excellent method of wavefront sensing in astronomical AO. Many variations exist, involving moving diffraction gratings and variable shearing.

### 2.4.2 Wavefront Curvature Sensing

Curvature sensing looks at the second derivative of the wavefront rather than the first as in the Shack-Hartmann method. This zonal sensing determines the curvature of the wavefront according to these slope measurements. Curvature sensors will take slope data from four adjacent subapertures as shown in Figure 2.7.

Each of the sections of the image are subapertures where the tilt in the $x$ and $y$ directions have been measured. With this information, the difference between the slopes of adjacent subapertures will relate to the second derivative. The cylindrical curvature about the $x$ and $y$ directions is given
2.4 Other Wavefront Sensing Techniques

Figure 2.7 Wavefront curvature sensing. Each square is a subaperture (Hardy 1998)

by

\[ C_x = \frac{\partial^2 z}{\partial x^2} = c[(x_1 - x_2) + (x_4 - x_3)] \quad (2.17) \]

\[ C_y = \frac{\partial^2 z}{\partial y^2} = c[(y_1 - y_4) + (y_2 - y_3)] \quad (2.18) \]

where \( c \) is a constant of proportionality. It is also possible to determine the cylindrical curvature along the 45° axes.

The cylindrical curvature about the \( x \) and \( y \) directions is given by

\[ C_{x+y} = c[(x_1 - x_3) + (y_1 - y_3)] \quad (2.19) \]

\[ C_{x-y} = c[(x_4 - x_2) + (y_2 - y_4)] \quad (2.20) \]

With these equations, coming from 8 measurements, we are able to represent orthogonal wavefront aberrations, such as defocus and astigmatism (Hardy 1998). The benefits of curvature sensing include being able to make measurements of four adjacent subaperture tilt errors at the same time. The photon noise error in the measurements is of the same order of magnitude of SHWFS.

However, there are limitations to curvature sensing. Error propagation in the reconstructor process of curvature sensors is much greater. This causes the error in curvature sensing measurements to increase as the square of the size of the full aperture, rather than logarithmically as is the case for tilt sensing. Because of this, curvature wavefront sensing is useful for small telescopes up to 6m (Hardy 1998).
2.4 Other Wavefront Sensing Techniques

2.4.3 Pyramid Wavefront Sensing

A pyramid sensor is similar to SHWFS through its use of a bicell detector. However, unlike the lenslet array of a SHWFS breaking the up the light in collimated space, a pyramid sensor uses a prism to break the light up at the focal plane of the beam. Each of these beams will land on a pixel of the detector.

Figure 2.8 shows a focused beam broken up by a pyramid. For an ideal point source each of the beams of broken up light is of equal intensity. When the image moves on the prism, the intensity on the CCD will shift. The detector will then calculate the normalized intensity differences to correct the image accordingly.

A benefit of the pyramid sensor is that the spatial resolution of the sensor is of the size of the pixel of the detector, rather than the size of an entire subaperture of multiple pixels for the SHWFS. There is a severe limitation in that the sensor becomes nonlinear quickly for large aberrations.

Figure 2.8 Diagram of a pyramid wavefront sensor. A prism at the image plane breaks the image up into 4 beams of light on a detector Credit: Cerro Tololo Inter-American Observatory
Chapter 3

Design of the KAPAO Wavefront Sensor

Chapter 2 discussed the basic theory of a Shack-Hartmann WFS. This wavefront sensing technique is relatively straightforward. However, in practice implementing the technique requires certain modifications to the basic method, depending on characteristics of the AO system. Figure 3.1 shows the layout of the testbed system from the DM to the WFS.

This chapter will discuss implementing the WFS on KAPAO, beginning with a basic model at the telescope primary and modifying this model to work within KAPAO using these three optics.

3.1 Breaking Up the Wavefront

When observing a field of stars, light from all the sources will enter the telescope primary mirror as overlapping collimated beams. It is at this point that correcting the wavefront for a chosen guide star will correct each of the sources because of this overlap.

Looking at a model of the telescope primary in Figure 3.2, we can see how the light entering the telescope needs to be broken up at the primary by an overlay of the 97 subapertures onto the mirror. The size of a subaperture for the SH-WFS algorithm is determined by Equation 2.7. This is the upper limit of subaperture size determined mainly by the Fried Parameter, \( r_0 \), which is the
3.1 Breaking Up the Wavefront

Figure 3.1 The testbed system with the WFS leg portion in a red box, starting from the DM going through various optics. The beam path is marked by the red arrows. The simplest SH-WFS implementation uses the beam right after the beam splitter, but for KAPAO it is necessary to use reimagining optics.

The largest aperture on a telescope allowed before atmospheric aberration begins to seriously affect seeing across said aperture. For the current WFS leg for the final system, the size of a subaperture is 9.36 cm.

Breaking the light up at the telescope primary is not feasible, so creating another pupil plane is necessary. As discussed in Section 2.1.2, pupil planes within the optical system are directly related to the telescope primary mirror. Therefore, the wavefront should be broken into subapertures at a pupil plane, to directly correlate to this mirror.

Using another optic, we can recollimate the light entering the telescope primary in order to form another pupil plane, where it is possible to break up the light. Figure 3.3 shows a model of a single subaperture on the telescope, one of the squares cut out of Figure 3.2, being recollimated. A
3.2 Scaling Between the Telescope and the Lenslet Array

Recollimating the beam is necessary in order to form a convenient pupil plane to break up the wavefront. However, this is not the only parameter for the recollimated beam, as there is also the size of the detector to consider and how these reimaged subapertures from the lenslet array map onto the CCD.

The system uses a SciMeasure Little Joe Camera with a low-noise 80x80 E2V CCD39. Fig-
3.2 Scaling Between the Telescope and the Lenslet Array

Figure 3.3 The simplest model of a SH-WFS on the telescope. This model is only looking at a single subaperture, beginning from the telescope primary, for simplicity. Distances between optics are not to scale.

Figure 3.4 shows the subaperture map on top of the 80x80 pixel array. The Robo-AO Software uses a grid of 97 subapertures on the SciMeasure CCD, oriented as shown by the golden highlighted boxes. The pixels on the full 80x80 CCD are combined into 3x3 pixel sized bins with a single pixel edge around the entire image used as a buffer, forming a 26x26 pixel array. The pixels on the 80x80 CCD are 24 µm across. A single subaperture is 6 pixels across, giving a subaperture size of 144 µm at the camera CCD.

The TMO telescope primary is broken into an array of subapertures, 11 subapertures across (Figure 3.2). At the CCD, these 11 subapertures, each subaperture 144 µm across, form a 1.584 mm diameter beam. The telescope primary is 1.029 m in diameter. This gives a magnification factor of 650. This means that the optics before the lenslet array, which has a 144 µm pitch that matches the subaperture size, must produce a collimated beam that is 650 times smaller than the beam entering the primary.
3.3 DM Magnification and Registration

One other factor to consider for the recollimating lens is the magnification and registration between the actuators on the DM and the subapertures on the CCD. From Figure 3.1, we can see that the DM is right before the WFS leg’s recollimating lens. The DM is in collimated space, and a set of optics must be used to remagnify this collimated beam so that it is the proper size at the lenslet array’s pupil plane, and by extension the camera CCD. The magnification must be correct such that the system achieves proper registration between DM actuators and subapertures. The correct registration relies on four DM actuators per subaperture. These actuators are located at the corners of the subaperture, as shown in Figure 3.5.

Our Boston Micromachines 140 actuator DM has a 400 µm actuator pitch. This means that the recollimating lens needs to project 400 µm onto a 144 µm sized area. The magnification factor

---

**Figure 3.4** An overlay of the subapertures on the 80x80 pixel grid. The golden squares are the 2x2 binned pixel subapertures. The smaller white squares are the pixels in the 26x26 pixel array (3x3 of the unbinned pixels). The subaperture index [0-96] correspond to the reconstructor matrix used in the software. *Credit: Alex Rudy’s PyAlign Software*
3.4 Imaging

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Figure 3.5 Subaperture mask on the CCD array with DM actuators labeled with green circles at the corners of the subapertures. The corner DM actuators are not used by the software. Credit: Alex Rudy’s PyAlign Software

is thus 0.36. Therefore, a set of optics must be used in order to achieve this magnification at the lenslet array pupil plane, so that the broken wavefront has the correct number of actuators for each subaperture at the CCD.

3.4 Imaging

A subaperture uses the 2x2 binned pixels as a bicell detector, described in Section 2.3. The lenslet array will break the wavefront into an array of focused spots that each land in the middle of a subaperture. Figure 3.6 continues expanding on our simplified model. Here the collimated beam of light after the collimating lens from Figure 3.3 is incident on an array of three lenslets. Conveniently, commercial lenslet arrays exist with a 144 μm pitch, the size of the subapertures on the CCD. This allows each microlens to correspond to a single subaperture. Because of this, the lenslet
Figure 3.6 A lenslet array focusing spots directly onto the detector. Looking at the detector on the right, the subapertures are 144 $\mu$m across, composed of 2x2 binned pixels or 6x6 unbinned pixels. The lenslet array has a 144 $\mu$m pitch, matching the subaperture size.

array images the spots onto the CCD with the proper separation between spots, rather than having to remagnify the spots to properly fall onto the detector.

Imaging directly onto the detector is not practical for KAPAO. The lenslet array has to be placed directly across from the CCD within the camera in order to directly image. This is not feasible because the lenslet arrays that KAPAO uses have extremely short focal lengths (4 mm and 8 mm). A device would have to be fabricated in order to hold the lenslet array at the appropriate distance from the CCD within the camera, such as a screwable mount that matches the threading of the camera head.

To overcome this issue, KAPAO uses a reimaging lens after the lenslet array. Figure 3.7 shows the optical diagram starting from the collimated space before the lenslet array to the WFS CCD. The magnification between the spots and the reimaged spots should be the 1:1. Therefore, according to the magnification of a thin lens, the reimaging lens should be placed a distance $2f_i$ from the focused spots and the reimaged spots will fall $2f_i$ from the lens, where $f_i$ is the focal length of the reimaging lens. This focal length is arbitrary and should be chosen to make the length of the WFS
optics leg reasonable. If the focal length is not properly set, then the spots will become defocused on the WFS CCD, which would produce larger spots within the CCD subapertures. This would improve the dynamic range of the detectors, at the cost of sensitivity when centroiding.

![Diagram of WFS leg](image)

**Figure 3.7** WFS leg from the recollimating lens to the CCD. The lenslet array, with focal length $f$, is reimaged onto the WFS CCD using a reimaging lens with focal length $f_l$ at twice the focal length to provide a $1:1$ magnification of the focal plane spots.

A final important factor in imaging the subapertures is the focal length of the lenslet array. As discussed in section 2.3.4, the maximum focal length for the lenslets can be found using Equation 2.8. Using a subaperture size of $144 \, \mu m$ and a central wavelength of $500$ nm, we find a maximum focal length of $10$ mm.

### 3.5 Design

We have discussed the modifications necessary in order to implement a SH-WFS into KAPAO. This includes a recollimating lens, followed by the lenslet array, and finally a reimaging lens to image the spots onto the CCD. With the parameters of each of these optics and the specifications for the Prime system in hand, it is now possible to design the SH-WFS leg for KAPAO Prime.

One of the factors to decide the optimality of the WFS leg is the focal length of the lenslet
array. This focal length determines the angular range of the subapertures. KAPAO currently owns two 144 µm pitch lenslet arrays, each of a different focal length. These focal lengths are 4.095 mm and 8.190 mm. The tolerance on the each focal length is ±5% of the nominal focal length, according to the vendor.

### 3.5.1 Recollimating and Reimaging Optics

KAPAO Prime’s design is based on off-axis paraboloid (OAP) mirrors, mirrors that work like lenses. We can find the proper focal length for the recollimating lens by using the focal length of the final OAP, right after the DM, and the magnification factor of 0.36, found in Section 3.3. The focal length of the OAP is 213 mm. With a magnification factor of 0.36, this gives a lens focal length of 76.7 mm in order to achieve the correct registration between the DM and the subapertures.

The major limitation for reimaging lens is the overall length of the WFS leg. The system currently uses a 20 mm focal length lens, which is a convenient distance for the size of the leg. This distance places the lens very close to the front of the WFS camera, giving a limited amount of working room after the optic.

### 3.5.2 Error in the Recollimating Lens and DM Magnification

Lenses with a focal length around the required 76.7 mm for the recollimating lens are commercially available from a variety of manufacturers. Edmund Optics offers a 10.0 mm diameter 75.0 mm focal length lens for $38 with a focal length tolerance of ±1%. Using this lens the magnification error will be 2%. The Alpha WFS uses a recollimating lens with a 36 mm focal length after the final OAP. This OAP has a focal length of 101 mm. In this case the error is 1%. Alpha demonstrates that there is tolerance for the system to function adequately when there is a magnification misalignment.

By adjusting the recollimating lens in the current Alpha WFS leg we can test the limits of
closed loop behavior when the registration between the DM and the WFS is misaligned because of magnification. Results were from looking at the spot quality on the Andor iXon 888 science camera that serves as the system's main imaging camera for the visible spectrum.

![Figure 3.8](image-url) Images from the scoring camera for a corrected spot. On the left, the WFS leg is aligned to the proper magnification for the reimaging lens, and on the right the reimaging lens is misaligned and moved forward toward the lenslet array 5 mm. There is noticeable degradation in the spot quality.

Figure 3.8 shows the difference between proper alignment and magnification misalignment using a point source. The misalignment was produced by moving the recollimating lens 5 mm toward the lenslet array. This magnification error is about 5%. We begin to see a noticeable loss of quality in the spot at this magnification error. The spot becomes dimmer and more spread out. The error from using a 75 mm lens as opposed to a 76.6 mm lens causes negligible issues compared to this 5 mm movement. The system should still be able to function and enter closed loop.

These tests must also take into account that the shift in lens position also contributes to error in the position of the pupil after the recollimating lens. This position error is negligible for these tests, since we are not dealing with an off-axis source. There have also been experiments performed around the pupil plane of the TT mirror that show that the displacement is within a fraction of a subaperture for adjustments slightly outside the pupil plane. To keep the lenslet array close to the ideal pupil location requires moving the entire WFS leg forward. However, these tests demonstrate that despite these errors, the system is still able to enter a stable closed loop. The quality of the correction will develop issues with further misalignment.
3.5.3 The Lenslet Array and Angular Range

Simplified Telescope and System Model

Figure 3.9 Simplified model of a single subaperture of the telescope through the system. Dashed lines are the chief rays from an on-axis source (red) and an off-axis source (green). The pupil and focal planes of the system have also been labeled, as well as the subaperture size, $d$.

When discussing the angular range of a subaperture, one needs to consider the translation of a tilt at the telescope primary to a tilt at the subaperture. Figure 3.9 shows a comparison between an on-axis source (red) and an off-axis source (green) going through the system to the subaperture at the final focal plane. The lenslet array is at a pupil plane, conjugate to the subaperture at the primary.

Figure 3.10 is a further simplified model that only focuses on the chief rays and angles between
3.5 Design

Figure 3.10 Further simplified model of a single subaperture of the telescope through the system, focusing solely on the chief rays from an on-axis source (red) and an off-axis source (green). The angles between the sources are labeled at the objective and at the lenslet.

The sources. We can find the angle between the sources at the subaperture using simple trigonometry with the dimensions of a lenslet. For a microlens of focal length $f_l$ and a subaperture of size $d$, the angle $\theta$ is given by

$$\theta = \tan^{-1}\left(\frac{d}{2f_l}\right)$$  \hspace{1cm} (3.1)

It is also convenient to look at the angular magnification between the objective and the lenslet array. Assuming the angles are small we can use the small angle approximation to show that

$$\frac{\theta}{\alpha} = \left(\frac{d}{2f_l}\right) = \frac{f_o}{f_l} = \frac{d_o}{d_l}$$  \hspace{1cm} (3.2)

where $f_o$ is the focal length of the objective lens. We can see that this is Equation 2.1. Now with this magnification factor we can translate the angular size of the subaperture depending on the
lenslet focal length to the angular size of the subaperture on-sky.

Field of View

![Focal Planes Diagram](image)

**Figure 3.11** The solid red lines are the central chief rays for on-axis sources in each of the subapertures. The green dashed lines are the central chief rays for sources coming in at an angle. We can see in the subaperture diagram to the right that for the same angle of incidence for the off-axis source, longer focal lengths will give a greater displacement within the subaperture, compared to a central on-axis spot. A longer focal length gives a narrower limit for the maximum tilt of the source.

As stated earlier, in the design of the WFS leg the easiest factor to adjust is the focal length of the lenslet array. This can alter the field of view (FOV) of each of the subapertures. As stated in section 2.3.3, the dynamic range of the detector is limited by the size of the spot. Altering the FOV can increase the angular range and shrink the size of the spot (larger FOV), or decrease the angular range and increase the size of the spot (smaller FOV). Increasing the angular size of a subaperture can improve the limit of the threshold between adjacent subapertures, such that adjacent spots do not fall into neighboring subapertures for a large tilt error. It is also possible to use shorter FOV lenslet array to produce a larger spot. This is useful in order to achieve a better dynamic range across the bicell and gain angular resolution. Note that angular resolution of the subaperture and
### Design

<table>
<thead>
<tr>
<th>Focal Length (mm)</th>
<th>Subaperture Size (µm)</th>
<th>Spot Width (&quot;')</th>
<th>Subaperture Size (&quot;')</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.095</td>
<td>144</td>
<td>1.80</td>
<td>11.16</td>
</tr>
<tr>
<td>8.190</td>
<td>144</td>
<td>3.60</td>
<td>5.58</td>
</tr>
</tbody>
</table>

**Table 3.1** On-sky angular sizes for the subapertures using each of the lenslet arrays. A wavelength of 400 nm was used to calculate the spot widths.

angular range of the subaperture are different and inversely related.

We can see from Figure 3.11 that for shorter focal lengths the system achieves a higher tolerance for tilt before an off-axis beam leaves the subaperture and the system suffers from crosstalk between adjacent subapertures. Table 3.1 shows the angular size of a spot and angular size of a subaperture for the 4 mm and 8 mm lenslet arrays. We can see that we achieve a greater FOV using the 4 mm focal length lenslet array. We also achieve a larger spot using the 8 mm lenslet array, at the cost of a smaller field of view. For the current wavefront sensor, it is more useful to use the 4 mm focal length lenslet array in the default setup, as the larger field of view will be more beneficial for worse seeing conditions.

**Experimental Verification of Lenslet Array FOV**

It is possible to experimentally verify the results that were calculated for the lenslet array FOV using the tip-tilt (TT) mirror within the system. The piezoelectric TT mirror is a Physik Instruments product. By manipulating the voltage across the crystals within the stage, we can produce a tilt and deflect the beam. The purpose of the mirror is to handle the lower order aberrations of tip and tilt across the entire aperture. Where the DM takes care of higher order aberrations and acts as a tweeter, the TT mirror acts as the woofer handling the lower order aberrations. It has a full angular range of 3.5 milliradians across 140 volts. This gives a resolution of about 5" per volt.

Experiments were performed on the Alpha system. It is possible to manually adjust the TT mirror and see the effect through a visual representation of subaperture centroids using an IDL
3.5 Design

Figure 3.12 A central 3x3 array of spots visualized using an IDL script. Centroids are marked with a blue plus sign and the red lines show the deviation from center. Here we see the difference between centered spots (left) and spots at the edge of the subapertures (right). Some of the spots have crossed over to the adjacent subaperture. Credit: Erik Littleton’s IDL routines.

The TT mirror is placed at the pupil after the first OAP in the system. The beam size at this point was measured to be $18.0 \pm 0.5$ mm. With a beam size of 1.584 mm at the lenslet array, these values give a magnification factor of 11.4. For the 8 mm lenslet array, the angular limit of the subaperture from a centered spot is 1813", using Equation 3.1. This translates to 159" at the TT mirror using the magnification factor. At about 5" per volt, a centered spot leaves the subaperture at 32 Volts.

By manually adjusting the TT mirror voltage, we drove the centered actuator spots to the edge of the subaperture. Repeating this measurement and taking an average, we determined that the spots reached the edge at $31.5 \pm 1.5$ V, where the uncertainty comes from the range of the measurements. The value of 32 V that was calculated is within the uncertainty of the experimental value. This verifies the predictions about the angular range of the 8 mm focal length lenslet array. Testing the 4 mm lens requires switching the lenslet array and realigning the reimaging lens and the SciMeasure camera. This is a test that can be performed in the future.
Chapter 4

Adaptive Wavefront Sensing

The previous chapter built a wavefront sensor for the KAPAO system from the ground up, highlighting all of the modifications necessary to implement a Shack-Hartmann wavefront sensor into the system. This chapter is devoted to analyzing variations to this basic KAPAO wavefront sensor leg that improve flexibility. The KAPAO wavefront sensor is currently designed to work around average conditions, with a Fried parameter of 9 cm. A question to ask is what could be modified in the system for it to adapt to different conditions. Constructing an adaptive wavefront sensor, a wavefront sensor that is modifiable for different situations, allows us to operate under a broader range of conditions. Table 4.1 summarizes the modifications and benefits under specific conditions.

In Chapter 3 we explored the constraints of the 4 mm and 8 mm focal length lenslet arrays. In order to maximize the field of view, the 4 mm focal length lenslet array was selected as a default. However, if the seeing conditions are better than average, the smaller field of view of the 8 mm focal length lenslet array would be beneficial. We can implement a WFS leg that could easily switch between each of these lenslet arrays, with the correct hardware. Another adaptive method is to change the size of the lenslet array completely. This changes the size of the subapertures at the telescope. The size of a subaperture at the telescope primary is currently set to 9.36 cm. Doubling the size of the subapertures increases the number of photons in each subaperture, increasing the
## 4.1 Swappable Lenslet Arrays

Chapter 3 discussed the difference in field of view (FOV) for each of the 144 \( \mu \text{m} \) pitch 4 mm and 8 mm focal length lenslet arrays that KAPAO uses. Table 3.1 lists the different spot sizes and FOV’s for each lenslet focal length. A comparison between the 4 mm and 8 mm focal length lenslet arrays reveals that the former provides a larger FOV, but sacrifices angular resolution. Because of the larger field of view, the 4 mm focal length lenslet array was chosen for the default configuration. There are situations, however, where the larger angular resolution of the 8 mm focal length lens is preferable.

The 8 mm focal length lenslet array produces twice as large a spot on the detector as the 4 mm focal length lenslet array. This increased spot size places a more uniform intensity distribution across the subaperture, thus giving the subaperture a better dynamic range in its corrections. The

### Table 4.1 Summary of adaptive wavefront sensing methods and the benefits that each of these changes would contribute.

<table>
<thead>
<tr>
<th>Star Brightness (Average)</th>
<th>Worse</th>
<th>Seeing (Average)</th>
<th>Better</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faint</td>
<td>Less Subapertures: Translates to more photons per subaperture; Compensate for fainter guide stars</td>
<td>Shorter Focal Length: Decreases Angular magnification; Provides larger FOV</td>
<td>Default</td>
</tr>
<tr>
<td>Bright</td>
<td>More Subapertures: Translates to fewer photons per subaperture; Better spatial resolution</td>
<td>Longer Focal Length: Increases angular magnification; Provides smaller FOV</td>
<td></td>
</tr>
</tbody>
</table>
intensity distribution on the bicell will not entirely enter one side of the detector as quickly as with a smaller spot (See the discussion of a narrow gaussian vs. a wide gaussian in Section 5.1.1). This allows for finer corrections on nights when the seeing is much less than the angular range of the detector using the 4 mm focal length lenslet array.

Figure 4.1 WFS leg setup with the 4 mm focal length lenslet array (top), the 8 mm focal length lenslet array modified by moving the current reimaging optic and the camera in unison (middle), and the 8 mm focal length lenslet array modified by switching the reimaging optic and leaving the camera in a fixed position (bottom).

Figure 4.1 shows the changes necessary to the WFS leg in order to swap the lenslet arrays. For
the 8 mm focal length lenslet array, two possibilities exist. For one modification, the reimaging lens and the camera move away from the lenslet array by 4 mm if the same reimaging lens were used as with the 4 mm focal length lenslet array. The second possibility swaps out the reimaging lens for a 19 mm focal length lens. This keeps the distance between the lenslet array and the camera static at 84 mm. With an 8 mm lenslet array there are 76 mm of space for a reimaging optic. We would place the optic halfway at 38 mm from both the camera and the lenslet array. Taking half of these divided distances means we use a 19 mm focal length reimaging optic to maintain the 1:1 magnification ratio. Everything before the lenslet array can remain as is, including the recollimating lens. This is because the lenslet array must be at the pupil plane after the recollimating lens, which is a characteristic of the recollimating lens and does not change when swapping between the different lenslet arrays.

An adaptive system that swaps between two lenslet arrays requires at least two translation stages for the first setup and one stage for the second. In the first setup, the first translation stage performs the swap. The second stage moves the lens and camera as a pair to the proper distance away from the lenslet array. The distance between the lens and camera does not change. In the second set up, the translation stage changes between the lenslet array and the reimaging optic. The distances for each mode are preset.

We can investigate the requirements for the first setup. The lenslet arrays are each 9x11 mm in total size. A variety of 2 inch (50.8 mm) mounts exist that would accommodate both lenslet arrays. It would be convenient to hold both arrays within the same mount, for the sake of space on the optical board. This can be achieved by using a custom holder for both of the lenslet arrays. A diagram of this setup is shown in Figure 4.2. A translation stage with a range less than 2 inches used to switch between the lenslet arrays is sensible for this design.

The reimaging lens and camera pair only have to move 4 mm, so a translation stage with a travel less than 1 cm is sensible. The major limitation for the second translation stage is the SciMeasure
4.1 Swappable Lenslet Arrays

Figure 4.2 A mockup of a lens mount holding a custom lenslet array holder that contains both the 8 mm and the 4 mm lenslet. A translation stage swaps the two lenslets into the optical path.

camera, because of its bulk. The system uses a beam height of 3 inches. As shown in Figure 4.3, in order for the SciMeasure to be at the correct height for the beam, the bottom of the camera must be 21 mm above the optical board. This leaves little room for a translation stage beneath the camera.

Because of the height restriction on the SciMeasure, this lenslet array swapping model may be unfeasible. Various commercial translation stages exist, but it is difficult to find a stage with a height less than 20 mm. Also, it is not advisable to move the camera, since the placement of the camera and the lenslet array must be precise to within 24 µm, the width of a pixel on the CCD. Translation stages from ThorLabs exist with increments of 0.05 µm meet these precision requirements. In the end, developing a custom mount for the SciMeasure camera that allows for translation at the specific height is a solution.

The second setup involves leaving the camera fixed in the system and swapping the lenslet array and the reimaging lens on the same stage. Figure 4.4 shows the limiting dimensions, which
Figure 4.3 The major limitation for the second stage is the size of the SciMeasure camera. In order to achieve the proper height for the beam, the SciMeasure must be about 21 mm above the surface of the optical board.

Figure 4.4 Top down perspective for the translation stage holding the mount for both lenslet arrays and the corresponding reimaging optics for each array. The beam for each situation is shown.

are the lens mounts for the reimagging optics. The current mounts are 4 cm across. Assuming that these mounts can be placed adjacent to each other, this places the optical axis through the center of each lens 4 cm apart. The same lenslet array mount in Figure 4.2 can be used. We can see from
4.1 Swappable Lenslet Arrays

Figure 4.5 that the proper separation is possible within this mount for the lenslet arrays, as long as the centers of the arrays are slightly offset from the center of the optical axis. As long as the correct orientation of the 97 subapertures is achieved in both relays, this decentering is not important.

![Diagram showing separation between lenslet arrays](image)

**Figure 4.5** Separation between the lenslet arrays necessary within the specialized 2 inch lenslet holder (Dimensions are to scale). The 4 cm separation is possible within these dimensions, although the center of the arrays will not be on the optical axis. This is not important as long as the proper orientation for the necessary lenslet arrays is achieved.

This setup would be a simple addition to the system and it is more feasible than moving the camera, in addition to the other two optics. By leaving the camera fixed in the system, and carefully aligning each of the relays before the CCD to within the 24 μm pixel size, it would be simple to achieve the correct alignment with a translation stage with increments of 0.05 μm. There is also the possibility of only swapping the lenslet array and simply leaving a 38 mm static reimaging lens. This would cause a magnification error for the spots on the CCD, making the spots larger within the subapertures. As mentioned in Chapter 3, larger spots could improve the detectors dynamic range at the cost of sensitivity.
4.2 Effects of Subaperture Size

Creating a mechanism used to switch between same sized lenslets of different focal lengths is a simple modification. There is another change that involves changing the size of the subapertures.

An array of larger subapertures has some benefits. Larger subapertures collect more photons. This is a benefit when observing fainter stars, as collecting more photons per subaperture will allow for correction of these fainter objects. Using a subaperture that is twice as large captures twice as many photons per subaperture making use of these fainter objects as guide stars more feasible.

This is also beneficial on nights with better than average seeing. The Fried parameter, $r_0$, is a measure of the atmospheric disturbance across an aperture and is what determines the size of the subapertures. The current value of about 9 cm is an estimation of the average seeing conditions at TMO. In reality, depending on the location, $r_0$ varies from 5 cm during strong turbulence to 20 cm for a night with good seeing, so particular nights could allow for a subaperture as large as 20 cm (Hardy 1998). With good seeing conditions, higher spatial resolution is not necessary, allowing for less computationally intensive calculations.

Using simulation software, previous work on KAPAO has studied different subaperture configurations. In Figure 4.6 we see two subaperture configurations compared. The 9x9 array uses larger subapertures across the same sized pupil as the 11x11 array. Simulations using the open source Yorick AO (YAO) software were ran using a 9x9 subaperture array and an 11x11 subaperture array, shown in Figure 4.7 (Rigaut 2011; Rudy 2011). These simulations demonstrated that there was little to be gained by using a 9x9 subaperture array. It may be necessary to revisit these simulations, considering the faint end of the plot is worse for larger subapertures.

Doubling the size of the subapertures to produce a 5x5 subaperture array is worth investigating for benefits under good seeing conditions. The following sections will investigate the implementation of an extreme case, where the size of the subapertures are doubled.
4.3 Doubling the Size of the Subapertures

Figure 4.8 compares the current 11x11 array to the new doubled 5x5 subaperture array at the telescope primary. This is one layout possibility that maximizes the number of subapertures across the array. This change could be made in one of two ways, either by shrinking the size of the pupil planes within the system, or by using larger lenslets that are 288 µm across to directly correspond to the telescope. Since the original size of the beam is 11 subapertures across, doubling the subaperture size produces 5 complete doubled subapertures, with excess illumination. In Figure 4.8, we see that the new subaperture array is smaller in height and width.

This new array requires a number of modifications to the orientation of the beam on the CCD and the DM actuator mapping in either case. These changes are extensive and must happen in hardware, software, or a combination of both. We will layout three distinct possibilities. In the first case, the size of the lenslets would be changed and modifications would be made in software. The second case minimizes the size of the pupil planes within the system at the DM and at the lenslet...
4.3 Doubling the Size of the Subapertures

Figure 4.7 Strehl ratio vs. guide star magnitude for a 9x9 subaperture array (54x54 pixel CCD) and an 11x11 subaperture array (80x80 pixel CCD). This plot shows that a 9x9 subaperture array offers little in terms of better correction for fainter stars. These simulations were ran using the open source Yorick AO (YAO) simulation software. Credit: Alex Rudy

array, in order to match the new subaperture array to the current DM actuator and subaperture dimensions. The third case maintains the current pupil size at the DM, and slaves actuators together in software.
4.3 Doubling the Size of the Subapertures

Figure 4.8 Comparison between an 11x11 subaperture array (left) and a 5x5 subaperture array (right) at the telescope primary pupil plane. In this case, the original subapertures on the left have been doubled in size to form the array to the right. The subapertures in the 11x11 array are about 9.36 cm across while the subapertures in the 5x5 are 18.72 cm across. The 5x5 subaperture array pupil plane is 9.36 cm smaller in height and width than the 11x11 subaperture array pupil plane, at the telescope.

4.3.1 Mode A: Increasing the Size of the Lenslets

Doubling the size of the lenslets, and keeping the same spot size at the lenslet array would break up the pupil plane into the desired subaperture array. Assuming the lenslet size is the only modification it would be necessary to double the size of the subapertures in software. Doubling the subaperture size to 288 µm on the CCD requires using an array of 4x4 pixels rather than the current 2x2 pixels. This opens up possibilities to implement more complicated centroiding algorithms, because of the larger array of pixels in a subaperture. These modifications are discussed in more detail in Chapter 5.

At the DM, Figure 4.9 shows how the new subaperture array maps onto the DM actuators. The software will have to be changed such that, when calculating a correction factor using the reconstructor, the DM actuators near the corners of the subapertures are made to move in unison
4.3 Doubling the Size of the Subapertures

4.3.2 Mode B: Shrinking the Size of the Pupil Plane

The second method is to make the pupil smaller at the DM and the lenslet array. This will cause minimal changes at the CCD. Figure 4.10 shows one way that the new 5x5 array of subapertures could be oriented onto the original 11x11 144 µm subaperture pattern on the CCD. This new illumination pattern is convenient because it does not change the mapping of the subapertures on the CCD.

Changing the size of the pupil at the lenslet array would be a simple change of the recollimating lens at the beginning of the WFS leg. This change will depend on the optics leading up to the WFS from the DM.

The major issue in this modification is the shrinking of the pupil planes before the CCD. The
4.3 Doubling the Size of the Subapertures

4.3.3 Mode C: Maintaining the Pupil Size at the DM

The last modification is a combination of the previously discussed methods. Modifying the pupil size at the lenslet array, such that the new pupil is broken by a 5x5 array of the 144 µm lenslets is a simple modification and these spots will be imaged to the unchanged CCD subapertures. It is possible to implement this change and keep the illumination on the DM the same, so that the optics leading up to the DM will have to be modified to shrink the size of pupil plane from the telescope to the DM to correspond to the actuators in Figure 4.10. This could be the setup for a new system, constructed from scratch. This would not be a feasible implementation for the current system, as the optics should remain static and changing them out on a regular basis is not a trivial task.
4.3 Doubling the Size of the Subapertures

before the DM can remain static.

This modification will involve the same DM modifications as in Mode A. The pattern of the subaperture on the DM will be the same as in Figure 4.9. This will require the same DM actuators to be grouped together in the reconstructor, slaving corner groups of actuators together. This change will occur in software as opposed to hardware. This adaptive modification is thus more feasible for the current system, as it does not require a change of the optical prescription before the WFS leg.
Chapter 5

Wavefront Sensor Spot Detection

To this point, there has been a focus on the hardware components of the WFS. There are also a number of modifications that can be made in software to improve the accuracy and efficiency of the WFS, especially in the realm of spot detection. One major component of the WFS reconstructor configuration software is the slope table. The slope table is a list of values that serve as a mapping function between the intensity distribution of a spot on a subaperture, which is what the software calculates, and the actual tilt of the spot on the subaperture. The generation of a slope table will be discussed here.

There is also the possibility of modifying the software to use more complicated subaperture arrays in order to improve efficiency in closed loop. Chapter 4 discussed hardware changes for implementing larger subapertures. Using larger subapertures on the actual CCD allows for these more complicated detection arrays. Implementing these changes would be another large project in itself. This chapter will discuss these changes in some detail as a project in the far future of KAPAO.
5.1 Slope Table

In Section 2.3.2, we saw how the bicell detector performs a measurement. The measurement is a simple centroiding calculation, finding the position of the center of the intensity distribution across the pixels within the subaperture to find the position of the center of the distribution, as in Equations 2.2 and 2.3 for the x-direction and y-direction. The slope is then numerically integrated using matrix multiplication with a predetermined reconstructor matrix. This is how the correction factor for the DM is calculated.

This centroiding, however, is not an accurate measurement of the spot displacement. This assumes that there is a linear relationship between the bicell output and the tilt of a subaperture. This relationship is actually highly nonlinear, as shown in Figure 2.5. To find the actual tilt of the subaperture and the proper correction factor, a precalculated slope table is used. A slope table is an implementation of a mapping function. The file can be arbitrarily long, and the index of each line corresponds to a certain intensity displacement. The software will find the index in the slope linearization file that is closest in value to the calculated centroiding number. Then the software will take the actual value on that line, which is the corresponding tilt value according to a slope response function, and use it in the numerical integration. This section will discuss the creation of a slope table for each of the lenslet arrays used on KAPAO.

5.1.1 Slope Table Generation

A diffraction limited intensity distribution can be approximated as a gaussian function. Figure 5.1 shows a 3D gaussian function on the bicell detector.

A slope table was generated by plotting a gaussian function on a 2x2 array. The intensity in each quadrant was added together and normalized according to the centroiding calculation in Equation 2.2. Then the gaussian was moved slightly in the x-direction, and the same calculation
Figure 5.1 A gaussian in 3-Dimensions centered on the bicell detector.

was performed. This movement was repeated across the array, and a set of intensity offset values corresponding to the displacement of the gaussian were determined. This was done for only a single axis; since the software assumes the intensity distribution is radially symmetric around the center of the distribution and it uses the same slope table for displacements in both the x and y directions.

The calculations were performed until the intensity distribution was displaced by $3\sigma$ from the center. This would reasonably end the slope table once 99.7% of the distribution was offset to one side of the detector. Any further measurement would not have any value, since once the distribution entirely leaves one half of the detector, the limit on the sensitivity of the detector has been reached.

In Figure 5.2 we can see the effect of the width of the gaussian on the generated slope function and how more narrow gaussians can produce a more nonlinear slope response function. As the gaussian functions take the same displacement from centered, the narrow gaussian will entirely
Figure 5.2 Slope response function for a narrow gaussian (blue dashed line) and a wide gaussian (green dashed line) plotted against a step function (orange dashed line) and a line of slope 1 (solid blue line) (left). The corresponding density plots for the gaussians are also shown, with the narrow gaussian on top and the wide gaussian on the bottom, both centered and taking the same sized step to the right (right).

enter one side of the detector more quickly than the wide gaussian. Imagine that the centroiding algorithm calculates a value of 0.8. If the system uses the narrower spot, then this corresponds to an actual slope displacement value of 0.15, looking at the graph in Figure 5.2. However, if the system uses the wider spot, this actual slope displacement would be closer to 0.37. By using a slope table that simply uses linear values, for a calculated value of 0.8, the system would use 0.8 from the slope table. This is highly overestimating the error in the slope, giving reason for the importance of a slope table.

By plotting this series of gaussian functions, and taking the value of the displacement of the function from center with respect to the difference in intensity across the array, we are able to use these new displacement values in the configuration files. The software will then take these more accurate displacement numbers and use them in calculations.
5.1 Slope Table

5.1.2 Slope Tables For KAPAO

According to Equation 2.5, the width of a spot is determined by the wavelength of the light, the focal length and the size of the detector. Below Table 5.1 lists the spot width for each of our two lenslet arrays, at a 400 µm wavelength.

<table>
<thead>
<tr>
<th>Focal Length (mm)</th>
<th>Subaperture Size (µm)</th>
<th>Spot Width (µm)</th>
<th>Spot Width Normalized to Detector Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.095</td>
<td>144</td>
<td>22.75</td>
<td>0.16</td>
</tr>
<tr>
<td>8.190</td>
<td>144</td>
<td>45.50</td>
<td>0.32</td>
</tr>
</tbody>
</table>

**Table 5.1** Spot widths from the two lenslet arrays using light at a wavelength of 400 nm.

Assuming these spot widths correspond to a $3\sigma$ radius around the mean of a gaussian distribution, we use a third of this value as the standard deviation of our gaussian distribution that will simulate the spot on the subaperture. Plotting this gaussian and moving it across the detector, we generate the slope response functions shown in Figure 5.3.

From this generated table of values, we take the displacement at regular intervals of intensity flux change to be the values in the slope table. With this slope table the software is able to act much more efficiently in its correction. When using the linear table, the software was underestimating the tilt error in the intensity distribution displacement. By underestimating these values in the mapping function, the software underestimates the correction factor and takes more steps to achieve convergence at a slope of zero. With these values, the software is able to accurately determine the values for the slope that it needs to correct and achieve convergence more quickly.

It will be a useful exercise to implement these slope tables for on-sky observations and compare the efficiency of each table. It may be that there are errors in alignment that would cause a mismatch between the spot size and the slope table. However these errors are minimal (much less than 1 mm in alignment). Assuming that the greatest error is in the distance between the optics, for
Figure 5.3 Slope response functions generated using a Python script. The orange line corresponds to the 4 mm lenslet array, and the blue line corresponds to the 8 mm lenslet array. The green line is a sample gaussian function with a standard deviation of 0.25 units of distance normalized to a pixel. A line is plotted for reference.

an error of 0.25 mm in the alignment, the percent error carried through to the spot width is about 1 µm.

5.2 Subaperture Modifications

Chapter 4 discussed modifications to the hardware of the WFS. This section will discuss the possibility of larger subaperatures on the CCD.

Continuing with the 5x5 subaperture array, Section 4.2 gave a solution for the layout of the
new larger subapertures onto a smaller portion of the original 11x11 subaperture configuration. Another solution is to use the same area of the CCD that the 11x11 subaperture array uses with the new 5x5 subapertures. Each of the subapertures in the 5x5 array would fall onto 2x2 of the original CCD subapertures. This provides 4x4 pixels for each of the new subapertures.

A bicell’s dynamic range is limited to a single wave of tilt. Once a single wave of tilt is introduced, the central core of the spot has been completely displaced to one side of the detector and there is no further change in the output of the detector. It is possible to implement a larger array of pixels to make a more sensitive subaperture with better linearity. One implementation is a 4x4 array of pixels, or a 2x2 array of subapertures. Arrays larger than 4x4 are difficult to justify (Hardy 1998).

Having a larger array of pixels opens the opportunity to increase the complexity of the measurements. A 4x4 array of pixels is shown in Figure 5.4. In this implementation we have grouped together a 2x2 array of subapertures. There are now actuators at the sides and in the center of this composite subaperture.

![Figure 5.4](image)

**Figure 5.4** A 4x4 array of pixels, where the dashed lines represent divisions between pixels and the solid lines represent divisions between the original subapertures. The green dots are DM actuators.

The standard calculation to determine the centroid of the spot on the array would be slightly
5.2 Subaperture Modifications

different from the bicell detector calculation. For a calculation in the x-direction, the array would be broken up into four sections, each section composed of the 4 pixels orthogonal to the direction of the x-axis. These sections are summed into binned values of $a_1$ through $a_4$. The slope is found by summing these sections across the detector as

$$A = \frac{w_1a_1 + w_2a_2 + w_3a_3 + w_4a_4}{a_1 + a_2 + a_3 + a_4}$$  \hspace{1cm} (5.1)

where $w_1$ through $w_4$ are weighting factors. This is one area where the 4x4 detector array can offer large benefits in dynamic range. By weighting arrays of pixels differently the detector array can obtain a more linear dynamic response function with better dynamic range. One method would be to weight the outer ring of twelve pixels higher than the center pixels. This would increase the response of the software for extreme aberrations. This weighting is visualized in the pixels in Figure 5.5.

![Figure 5.5](image)

**Figure 5.5** The same 4x4 array of pixels, this time showing the pixels that are weighted higher in red.

This algorithm will offer benefits to detector sensitivity and the linearity of the response function. However, whether the algorithm is too computationally intensive to implement with current software and the current computer systems available is yet to be seen. There is also experimenta-
tion to be performed on what a proper weighting would be for the spot sizes used in KAPAO.

One major issue with using more pixels to the detector array would be pixel noise and error. For a generic detector the error in angular position is proportional to square root of the number of pixels in a detector array (Hardy 1998). This increased wavefront measurement noise is negligible for observing brighter sources, as the angular position error is also inversely proportional to the number of photons per subaperture.

On a final note, it is also possible to implement curvature detection with this array. By using the grid as an array of 2x2 of the original 144 µm subapertures, it is possible to take differences between adjacent slopes to obtain a higher order term for the correction algorithm. This curvature sensing requires a different algorithm and is extremely complex to perform with current hardware.
Appendix A

System Throughput

System throughput was determined using an optical power meter throughout the system and with individual components. This appendix highlights the throughput values, theoretical and experimental, within the system and of individual components. All experimental values were determined with a 635 nm laser. Theoretical values come from the various manufacturers of the optics.

A.1 Component Throughput Characterization

A.1.1 Reflectivity of the DM

The KAPAO project currently possesses three deformable mirrors. One of these is on loan from Sonoma State University. Figure A.1 shows the average throughput for each of these DM’s from measurements at roughly 45 degree angle of incidence. These measurements include the throughput lost from the DM’s protective window, a clear casing that shields the inner actuated mirror that contributes about 1% throughput loss.

Uncertainty for each of these mirrors is taken as the standard deviation of the measurements, coming out to about 1%. The unprotected gold and aluminum DM’s have reported throughput
Figure A.1 Throughput for the deformable mirrors. The protected silver coated DM and the gold coated DM have similar throughputs, and are much higher than the unprotected aluminum.

values of about 93% and 87% at about 635 nm, according to the manufacturer (See Figure A.2). This discrepancy between our experimental results and the manufacturer’s reported values is left for further investigation.

Figure A.2 Throughput for the deformable mirrors across a range of wavelengths according to the manufacturer. The angle of incidence is 45 degrees. Credit: ThorLabs
A.2 Theoretical Composite Throughput for Prime System

A.1.2 Throughput of the WFS Leg

<table>
<thead>
<tr>
<th>Optic</th>
<th>Throughput (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recollimating Lens (0.5&quot; optic)</td>
<td>98 ± 1</td>
</tr>
<tr>
<td>Lenslet Array</td>
<td>93 ± 1</td>
</tr>
<tr>
<td>Reimaging Lens (0.5&quot; optic)</td>
<td>96 ± 1</td>
</tr>
<tr>
<td>Total Throughput</td>
<td>87 ± 2</td>
</tr>
</tbody>
</table>

Table A.1 Throughput of each of the components of the WFS leg and the total throughput.

The WFS has a throughput of 87%. These tests were run on the testbed system, with the 4 mm focal length lenslet array. This throughput could be improved by coating the optics in the path, as they are currently uncoated.

A.2 Theoretical Composite Throughput for Prime System

Figure A.3 shows a graph of the composite throughput for KAPAO Prime, the final system. This chart assumes that all mirrors will be coated with a protected silver coating. The value for the lenslet array and the WFS are taken from the tests performed in the previous section. This means that the chart also assumes that the throughput of the lenslet array is 93% across this range of wavelengths, which has yet to be verified. We can see that the major losses occur at the DM and the WFS leg.
Figure A.3 Throughput for the Prime system, component by component, to the WFS. The values used for each component are taken from the reported values of the respective manufacturers for the wavelength range of interest for the WFS.
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