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Optimizing Flywheel Design for use as a Kinetic Energy Recovery System for a Bicycle

1. Introduction

A flywheel is an energy storage device that uses its significant moment of inertia to store energy by rotating. Flywheels have long been used to generate or maintain power and are most identified with the industrial age and the steam engine. In one sense it can be thought of as a rechargeable battery that store energy in the form of mechanical energy instead of electrochemical. Flywheels have been gaining popularity as a possible replacement for chemical batteries in vehicles, but until last year there was no record of a flywheels being used to increase the efficiency of a bicycle.

1.1 Motivation

In 2011, Maxwell von Stein, a student at Cooper Union, added a flywheel and a continuously variable transmission to his bike for his senior project.[1] He used a car flywheel he found that weighs 15 pounds. His idea won him the Nicholas Stefano Prize, which is Cooper Union's award for superior mechanical engineering design. He also gained quite a bit of notoriety on various biking websites and was featured in NPR's weekly segment, "Science Friday."[2] This idea of adding a flywheel to a bicycle is very appealing because it can increase the efficiency of what is already considered a very efficient machine. The only concern with Mr. von Stein's design is that his flywheel is very heavy. It was made for a car, so an extra 15 pounds would hardly be significant in such a heavy vehicle. However, when 15 pounds are added to a bike it makes a significant difference in the additional work it takes to accelerate the bike. Mr. von Stein estimated that the additional weight adds about ten percent more weight to the system, and he also estimated the peak efficiency gain from his flywheel is ten percent. This means at its peak, the flywheel is only making up for the efficiency lost by its additional weight. If the flywheel was optimized for the different design requirements of a bike, it could increase the efficiency of a bike in more significantly.

Figure 1. Maxwell von Stein's Flywheel bicycle

1.2 Flywheel Background

Rotating wheels have been used to store and deliver energy since prehistoric times. The potter's wheel is perhaps the first invention to resemble a flywheel and it has existed for 4,000 years. The first instance of the word flywheel occurred in 1784 during the industrial revolution.[3] At this time flywheels were used on steam engine boats and trains and were often used as energy accumulators in factories. Flywheels became more popular with steep drops in the cost of cast iron and cast steel. In the industrial revolution, flywheels were very large and heavy so that they could store significant energy at low rotational speeds. The first use of flywheels in road vehicles was in the gyrobuses in Switzerland during the 1950's. The flywheels used on the buses were 1500kg and had a diameter of 1.626 meters. When they were fully charged they could store $3.3*10^7$ Joules.[3] Flywheels are now found in many road vehicles as well as space, sea, and air vehicles. Flywheels are also used for energy storage in power plants and as voltage controllers. Newly raised concerns about the environment have increased interest in flywheels. This along with the developments in carbon fiber are making flywheels a viable technology despite the developments in battery technology. Flywheels now are smaller, can spin faster, are safer, and output more energy than ever before. In fact a group at the University of Texas in Austin has developed a flywheel that stores and outputs enough energy to take a train from a full stop to cruising speed.[4] Though flywheels are most identified with the large spinning wheels of the industrial age, the technology may currently be finding new life in a environmentally conscious world with major developments in materials science.

The distinctive feature of the flywheel is its high power density. Especially with the development of carbon fiber, small, light flywheels can store very high amounts of energy safely. They also can be charged very quickly and can deliver high amounts of power, as shown by their use in nuclear fission plants. Flywheels are also a zero pollution method of storing energy. In short bursts flywheels are near one hundred percent efficient. Though efficiency drops depending on how long the flywheel has to stay charged, the decay in efficiency depends greatly on the system the flywheel is placed in and whether or not it has any type of housing. The main limit for flywheel performance will be the efficiency of the transmission connected to the flywheel. Such ratings can cut the power transfer in half or by even more. Transmission design also often puts an upper limit on the speed at which the flywheel can spin, in some cases effectively putting an upper limit on its energy capacity. Since it is a mechanical system that stores energy, there are concerns about fatigue and wear from vibration and repeated use, as well as safety concerns. With modern advanced flywheels, they are usually designed to break so that there would be no large pieces, and would be contained in a housing which will prevent injury of anyone nearby.

In road vehicles, as well as other applications, flywheels are being considered as a replacement for electrochemical batteries.[3] Flywheels have high energy per kilogram, low charging times, are lightweight, and have a longer lifetime than batteries. Batteries need to be replaced during the life of a hybrid vehicle, which can be costly and hazardous to the environment. The main limit on flywheels in vehicles is the design and rating of the transmission. It is already clear how to create electrical energy from braking, but there aren't designs for a purely mechanical kinetic energy recovery system in cars. Flywheels do not yet have an economic advantage over batteries since flywheel technology in small road vehicles is not well developed, but there is a promising future in flywheel hybrid cars.

1.3 Flywheel use in cars

Flywheels have been used in cars for a very long time, but they haven't been used as kinetic energy restoration systems until recently. The flywheel's main use in cars is to convert the power from the engine and transfer it to the clutch plate.[5] An internal combustion engine generates its power by firing pistons. Only one in four of these strokes actually drive the vehicle. This means that the output power of the engine is not steady. This problem is less severe the more cylinders a car has, since the pistons will be firing at different times to make up for the long gaps. Regardless of how many cylinders are present a flywheel is still needed to maintain a steady power supply. All of the pistons firing drive the flywheel to create a smooth power output. The flywheel can then transfer its power by being pushed into the clutch plate. When the flywheel and the clutch plate are pushed together, the clutch plate starts spinning which engages the transmission. A typical set up for a flywheel in a car is shown below in figure 2.The flywheel is therefore a key part in delivering the power from the engine. Overall it serves three purposes; as energy storage for the engine, as a surface for the clutch plate and as the drive gear for the starter.

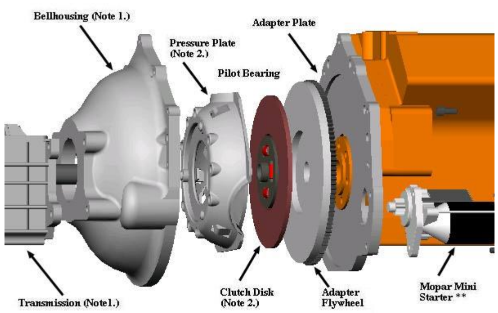


Figure 2. A basic diagram of how a flywheel connects the engine to the transmission

Figure 3. A picture of the Williams hybrid flywheel system.

Cars now have flywheels not just used to transfer power from the engine to the transmission, but as kinetic energy recovery systems. Formula One changed its rules in 2009 to allow kinetic energy recovery systems in racing cars.[6] These systems could either be electric or mechanically based. Some teams chose to develop flywheel based recovery systems. One such system that is becoming popular in racing cars is the Williams Hybrid Power flywheel system, shown in figure 3.[7] This system converts braking energy into electrical energy which is used to spin a carbon fiber flywheel. This flywheel can get up to speeds of 60,000 rpm, which on its outer rim is approximately twice the speed of sound. For this reason the flywheel is stored in a vacuum sealed chamber and suspended with magnetic bearings. This prevents any significant air resistance and the flywheel is thus very efficient. The system weighs approximately 300 pounds, which is fairly heavy for most cars, but the system can also be fitted to buses or subways, for which the weight gain would be much less significant.

Car manufacturers are also starting to make flywheel based recovery systems for commercial vehicles. Both Volvo and Jaguar are developing prototype hybrid cars that use flywheel technology.[8] Instead of driving the car for long distances, these systems will be used to give the car a boost when needed. A Jaguar representative admitted that the system would only be able to go about a half a mile on its own power. The power output in boost will still be significant though. Volvo, which expects its flywheel hybrid car to hit markets in 2015, claims that its kinetic energy recovery system will be able to give the car an 80 horsepower boost when engaged. With this kind of power there will be significant fuel savings.

1.4 Transmissions

Every mechanical system that needs to control the power it outputs requires a transmission. The transmission accepts the power input to the system and outputs it in whatever fashion is necessary for the system. In the case of the bicycle chains and sprockets are used to deliver the power from the crank to the back wheel. The relative size of the sprockets will determine how quickly the wheels spin relative to the crank. Chain drive is also used to transmit power between the flywheel and the bicycle. In Mr. von Stein's flywheel bicycle, he used a continuously variable transmission to connect the crank.[1] A continuously variable transmission is a device that changes the radius the chain rests on to continuously change the gear ratio. This allows the rider to shift the transmission from allowing the crank to spin quickly to allowing the flywheel to spin quickly. The mathematics of the gear ratio will be discussed in the next section. A variable transmission would suffice for the purposes of the flywheel. This would mean the flywheel has a fixed number of gear ratios it can switch between, much like bicycles that have more than one speed. The premise for a variable transmission is still the same as it is for the continuously variable; the gear ratio will switch between the crank spinning quickly and the flywheel spinning quickly. The design of the transmission is the limiting factor for the speed of the flywheel, and will largely determine how efficient the flywheel system is. Inefficiencies in the transmission as well as speed and torque limitations caused by the transmission will by and large outweigh any theoretical limitations of the flywheel.

2. Theory

2.1 Energy of Rotation

The energy stored in a flywheel is its rotational kinetic energy

$$E = \frac{1}{2} I\omega^2 \tag{1}$$

where ω is the rotational velocity and I is the moment of inertia, which is defined as

$$I = \int r^2\, dm = cmr^2 \tag{2}$$

where c is a constant determined by the mass distribution.[9] Substituting equation 2 into equation 1 we get

$$E = \frac{1}{2} cmr^2\omega^2 \tag{3}$$

Therefore we have four variables to consider for energy storage in the flywheel. The same amount of energy will be transferred to the flywheel no matter what the design is, so our choice of design will simply place the energy more in some variable than others. There is no downside to maximizing c, so a flywheel with a majority of its mass at maximum r is a must in the design.

The radius will be limited by our placement of the flywheel and the size of the bike. The rotational velocity will be limited by our transmission design. The cost of additional mass will be discussed in the next section.

2.2 Efficiency

The gain you get from a flywheel must be measured against the extra power required to move the bicycle from the extra weight of the flywheel. Extra work is needed to accelerate the bike because of the flywheel. Therefore the efficiency gained from the flywheel can be shown as

$$\varepsilon_{total} = \varepsilon_{gained} - \varepsilon_{lost} \tag{4}$$

The efficiency gained can be expressed as the energy stored in the flywheel (from the above section) over the total energy in the bike. The efficiency lost can be expressed as the energy required to push the extra weight of the bike over the total energy in the bike.

$$\varepsilon_{total} = \frac{E_{flywheel}}{E_{total}} - \frac{E_{fly\ accel}}{E_{total}} \tag{5}$$

After plugging in equation 3, we get

$$\varepsilon_{total} = \frac{\frac{1}{2}\eta c m_f \omega^2 r^2}{\frac{1}{2}m_{total}v^2} - \frac{\frac{1}{2}m_f v^2}{\frac{1}{2}m_{total}v^2} \tag{6}$$

where ε is the efficiency, η is the efficiency of the transmission, m_f is the mass of the flywheel, and v is the velocity of the bike. Canceling like terms we get

$$\varepsilon_{total} = \frac{\eta c m_f \omega^2 r^2}{m_{total}v^2} - \frac{m_f}{m_{total}} \tag{7}$$

With this in mind the flywheel design should minimize the mass of the flywheel in favor of a larger radius or faster speed, since the total efficiency will be much higher.

2.3 Critical stress

Another design consideration to take into account is the stress experienced by the flywheel. This is given by

$$\sigma = \frac{1}{2}\rho r^2 \omega^2 \tag{8}$$

where ρ is the density of the material.[3] Usually the key for material selection of a flywheel is the highest possible tensile strength over density. The maximum stress a bike can handle is the tensile strength, so

$$\sigma_{max} > \frac{1}{2}\rho r^2 \omega^2 \tag{9}$$

For a bike flywheel, the size and speed are not likely to even approach a higher enough value to create a stress above the tensile strength of a material, so this consideration is not so important. The tensile strength of standard steel is 250 MPa.[10] Since the bike radius will be less than one, and the angular velocity will be less than 100, the tensile strength will not be reached. Since bikes are light, it is much more important to the overall efficiency to minimize the mass of the flywheel. With this in mind the flywheel should have as large of a radius as possible, and spin as fast as possible. The limit to the radius is the physical limitations of the size of the bike and the practicality of fixing the flywheel to the bike. The issue with a high radial speed is the friction losses over time.

2.4 Transmission

It will be necessary to design a transmission to transfer energy from the bike to the flywheel and vice versa. The gear ratios will determine the properties of the transmission, and is measured in many different ways. The relative size of the gears will determine the maximum speed the flywheel will spin at relative to the bike. The gear ratio, R, is defined as

$$R = \frac{\omega_A}{\omega_B} = \frac{N_B}{N_A} = \frac{T_B}{T_A} = \frac{r_B}{r_A} \tag{10}$$

where ω is the rotational speed, N is the number of teeth in the gear, T is the torque, and r is the radius, and A and B refer to the two different gears.[11] A high gear ratio means that B will have high torque and A will have high speed. Therefore, a flywheel designed to operate at a high speed will need a high gear ratio transmission, considering the flywheel as A. For a flywheel that operates at low speeds but has a large radius, the gear ratio would need to be low since it will require more torque to spin a flywheel with a large radius. Also, the target speed for maximal efficiency must be taken into consideration. A flywheel that will be used to add a boost after a complete stop should have a high gear ratio because gear B should have high torque to get the bike wheels spinning again. However, a flywheel that will be used for an extra boost after slowing down, but still remaining at high speeds, the transmission should have a low gear ratio so that gear B can spin fast. If the flywheel were to have a large radius and be used for stop and go, or if the flywheel were to spin quickly and be used for a boost at high speeds, the gear ratio could be even to maximize torque in gears A and B, or to maximize speed in gears A and B.

2.5 Aerodynamic Drag on the Flywheel

The torque on both side of a thin disk flywheel of uniform thickness caused by the air is described by

$$T_{Air} = \rho_g \omega^2 r_o^5 C_m \tag{11}$$

where T_{Air} is the torque experienced by the wheel, ρ_g is the density of the air, ω, is the rotational speed of the wheel, r_o is the radius of the wheel, and C_m is a constant associated with the type of air flow the flywheel is experiencing.[3] C_m usually depends on the Mach number, the Knudson number and the Reynolds number, but in the case of the thin uniform thickness flywheel the Mach number can be ignored, and since the flywheel will be operating in atmospheric pressure, the Knudson number is low enough for the gas to be considered a continuous medium. This means in our case C_m only depends on the Reynolds number, as shown in the expression below

$$C_m = 3.87 R_e^{\frac{-1}{2}} \tag{12}$$

The Reynolds number is widely used in engineering. It is a dimensionless number that gives you the ratio of internal forces to viscous forces. It can be used to determine whether fluid flow will be laminar or turbulent. For our case, it can be expressed as,

$$R_e = \rho_g r_o^2 \omega / \eta \tag{13}$$

where η is the dynamic viscosity of gas, given by

$$\eta = \frac{1}{3} \rho_g \bar{v} \lambda \tag{14}$$

This expression is true if the equilibrium velocity distribution of the gas is Maxwellian, which is not the case. For real gases, the following expression is more accurate.

$$\eta \cong \frac{1}{2} \rho_g \bar{v} \lambda \tag{15}$$

Here $\bar{v}$ is the mean velocity and λ is the mean free path. The mean velocity can be expressed as,

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} \tag{16}$$

where K is the Boltzman constant, T is temperature, and m is the mass of the gas molecules. The mean free path can be expressed as,

$$\lambda = \frac{m}{\sqrt{2} a^2 \rho_g \pi} \tag{17}$$

where *m* is the same as above and *a* is the effective molecular diameter. The effective molecular diameter for air depends on N_2 and O_2, which have very similar values. The diameter changes slightly with temperature, but for the temperature values we will be dealing with the effective molecular diameter can be approximated as $3*10^{-10}$m.

2.6 Stability

Another consideration when adding a flywheel to a bike is the possible stability issues with the torque the flywheel will create. When making a turn this force could possibly push the bike into the ground. The expression for the gyroscopic moment is

$$M = I\Omega \times \omega \tag{18}$$

where M is the gyroscopic moment, I is the moment of inertia, Ω is the angular velocity caused by the bike, either by tilting or turning, that the flywheel is experiencing, and ω is the angular velocity of the rotation of the flywheel.[3] Whether or not this force is significant in the stability of the bike, it can easily be avoided by counter rotating flywheels. This stability problem should not be a factor since bike wheels themselves rotate and generate the same force and remain very stable when turning. It has been shown that the gyroscopic moment of a bike is not the reason for its stability. A bike was made with counter-rotating hoops to cancel out the gyroscopic moment of the wheels.[12] Tests were done on the bike when the hoops were rotating in the direction opposite of the wheels, with the wheels, and with no rotation at all. In all three cases the bike was rideable, which in our case shows that the added moment the flywheel contributes will not make the bike unrideable.

2.7 Statics of clutch design

The clutch that will be used for the flywheel will use a caliper to engage and disengage the two gears, which will shift the gear ratio. This mechanism will be explained more in the following chapter. A similar example problem exists in Shigley's *Mechanical Engineering Design*.[13] Though the example problem does not involve rotating velocities, it is similar enough to the clutch design in the bike that it is worth reproducing below.

The problem we are analyzing is shown in figure 4. The analysis of all friction clutches and brakes uses the same general procedure:

1. Assume or determine the distribution of pressure on the frictional surfaces.
2. Find a relation between the maximum pressure and the pressure at any point.
3. Apply conditions of static equilibrium to find (a) the actuating force, (b) the torque, and (c) the support reactions.

We will now apply these steps to our problem. Figure 4 shows a short shoe hinge at A having an actuating force F, a normal force N pushing the surfaces together, and a frictional force fN, f being the coefficient of friction. The shoe is stationary and the surface it is contacting is moving to the right. The pressure at any given point is designated as p and the maximum pressure is designated as p_a. The area of the shoe is A.

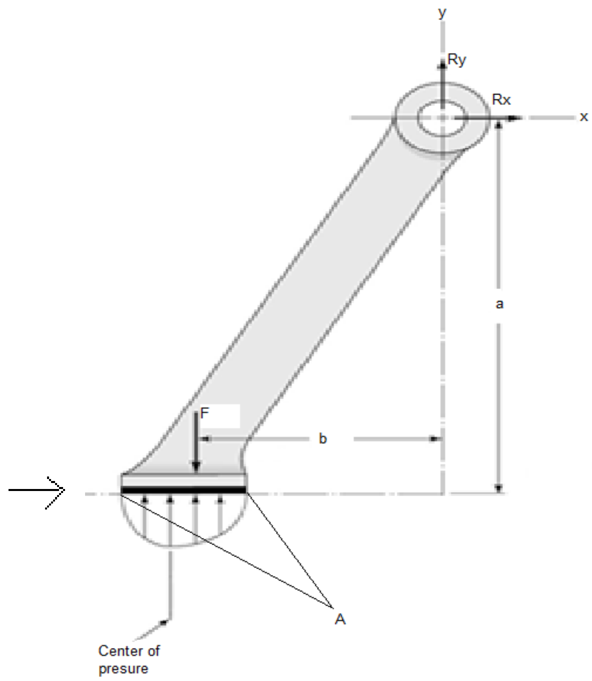


Figure 4. From Shigley's *Mechanical Engineering Design*, A shoe working as a brake or clutch

Since the shoe is short, we can assume the force is distributed evenly over the frictional area. From this assumption it follows that

$$p = p_a \tag{19}$$

i.e. the pressure at any point is equal to the maximum pressure. Since the pressure is evenly distributed, the normal force can be expressed as

$$N = \ p_a A \tag{20}$$

Now we apply the conditions of static equilibrium by taking the summation of the moments around the hinge pin.

$$\sum M_A = Fb - Nb + fNa \tag{21}$$

Substituting for N with equation 20 and solving for the actuating force F, we get

$$F = \frac{p_a A(b - fa)}{b} \tag{22}$$

Taking a summation of forces in the horizontal and vertical directions gives the hinge pins reactions:

$$\sum F_x = 0 \qquad R_x = f p_a A \tag{23}$$

$$\sum F_y = 0 \qquad R_y = p_a A - F \tag{24}$$

This completes our analysis of the problem.

2.8 Stress Analysis

Figure 5 below shows a general three-dimensional stress element showing three normal stresses and six shear stresses. If an element is in static equilibrium, then

$$\tau_{xy} = \tau_{yx} \quad \tau_{yz} = \tau_{zy} \quad \tau_{zx} = \tau_{xz} \tag{25}$$

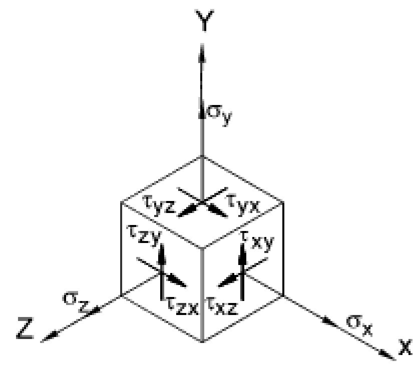


Figure 5. The principle stresses for a given three dimensional element

For shear stresses applied to a beam, the shear forces will cause a bending in the beam if it is not supported on one side. The shear force and bending moment are related by

$$V = \frac{dM}{dx} \tag{26}$$

Where V is the shear force and M is the bending moment. So if there is a force versus distance graph, them the bending moment would equal the area under the curve. With the bending moment, the bending stress can be obtained.

$$\sigma_B = \frac{My}{I_x} \tag{27}$$

Where σ_B is the bending stress, y is the perpendicular distance from the axis, and I_x is the second moment of area. Since the axle the flywheel system will be resting on is a hollow cylinder, the second moment of area can be expressed as,

$$I_x = \frac{\pi}{4}(r_o^4 - r_i^4) \tag{28}$$

where r_o is the outside radius and r_i is the inside radius. This turns equation 27 into

$$\sigma_B = \frac{4My}{\pi(r_o^4 - r_i^4)} \tag{29}$$

The maximum bending stress changes depending on the material. For steel it depends on the type used, but typically ranges from 20ksi to 40ksi.[10] Therefore the shear force cannot be so great as to create a bending stress that exceeds approximately 20 ksi. The measure of a design's safety is called the safety factor.

$$Safety\ Factor = \frac{Material\ Strength}{Design\ Load} \tag{30}$$

A safety factor of one will not be able to handle any additional stress other than the theoretical stresses from design. A safety factor of 2 would be able to handle double the design stress. A safety factor of less than one is not at all acceptable in design.

2.9 Rotor Dynamics

The flywheel will need to be attached to a rotor to receive power from and transfer its power to the bike. Rotors have one or multiple critical speeds where their deformation goes to infinity and they don't work. The deformation of a rotor is expressed as

$$D = \frac{u}{(\frac{k}{m\omega^2} - 1)} = \frac{u}{((\frac{\omega_{cr}}{\omega})^2 - 1)} \tag{31}$$

where D is the deformation, *u* is the eccentricity, and *k* is the stiffness.[3] The critical speed for a rotor is expressed as

$$\omega_{cr} = \sqrt{\frac{k}{m}} \tag{32}$$

Since the stiffness of steel is 200GPa and the rotor will be on the order of unity in kg, a bike is most likely not going to reach the critical velocity. A rotor can operate above and below the critical velocity safely, but at the critical velocity the rotor will vibrate strongly and may cause damage. Therefore the rotor dynamics will always be in the subcritical range. This means the rotor will have somewhat high levels of vibration and the bearings will have more drag.

2.10 Aerodynamic Drag on the Bike

The air resistance varies quadratically with the speed of the bike at high velocities. At speeds around 10mph or more, it is even a fair approximation to ignore friction as a negligible force.[9] For a bike coasting, the equation of motion is

$$m\frac{dv}{dt} = -cv^2 \tag{33}$$

$$m\frac{dv}{v^2} = -cdt \tag{34}$$

where c is a coefficient based on the cross sectional area of the object and property of the fluid it is in. Integrating equation 28, we get

$$m \int_{v_0}^{v} \frac{dv'}{v'^2} = -c \int_0^t dt' \tag{35}$$

$$m \left(\frac{1}{v_0} - \frac{1}{v} \right) = -ct \tag{36}$$

Solving for v,

$$v(t) = \frac{v_0}{1 + c v_0 t/m} \tag{37}$$

Equation 31 expresses how the velocity of a bike decreases due to air resistance when coasting

2.11 Implications for Final Design

With all of these considerations taken into account, the mass of the flywheel should be as small as possible and should be distributed as much as possible on the outer radius, then the radius should be made as large as can fit on the bike, and the rest of the energy will be in the rotational speed, which hopefully is not too high to get significant friction losses.

3 Design

3.1 Goal

The transmission for the flywheel should be simple enough to make without a wealth of machine shop experience and should be efficiently allow the flywheel to build up energy and release its energy. The clutch should not be an on/off mechanism, but instead should switch between the bike's crank spinning quickly and the flywheel spinning quickly. An on/off mechanism would switch between the flywheel being engaged with the bike, or being disengaged with the bike. This would mean that the bike could either gain or dispense its energy when it is engaged with the bike, and do nothing but lose energy to friction when disengaged. A clutch system that switches between the flywheel and the crank spin speeds however has a clear switch between the flywheel accumulating energy and releasing energy. To accomplish this, a two speed transmission will be set up between the crank and the flywheel to allow two very different speeds for the flywheel to turn. For simplicity, the flywheel itself will be a smaller 20 inch steel bike rim. This allows for an off the shelf part that can easily be attached to a bike axle.

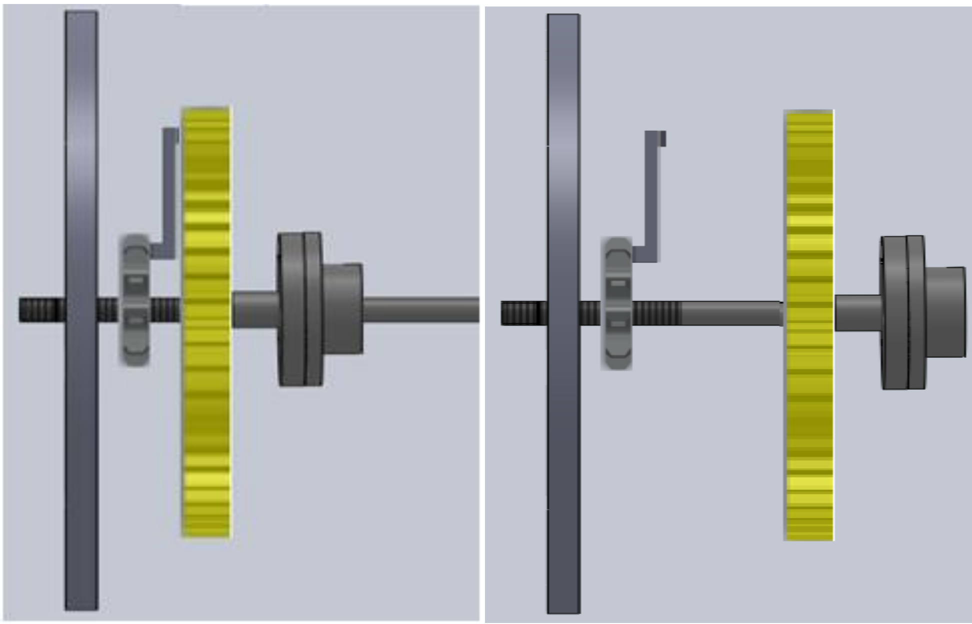
3.2 Clutch and Shifting Mechanism

The clutch is the most complicated and troublesome aspect of the flywheel system design. Mr. von Stein's implementation used a continuously variable transmission. This is a device that can seamless change its shape to create an infinite number of gears ratios. The device is simple to implement but is very complicated to make, so it is not within the limits of this project. A normal multi-speed bike has devices such as derailers to switch between its sprockets, but the shift in sprocket size is considerably smaller than what the clutch for the flywheel will switch between. The flywheel's clutch will shift between a gear much smaller than the crank sprocket to a gear much larger than the crank sprocket. This is too difficult of a task to physically move the chain from one sprocket to the other. A disc or caliper braking system might be thought of as a simple, off the shelf clutch, but what they have in their simplicity they suffer in the difficulty of implementation. All bike braking systems of those types use a hydraulic cable to actuate the brakes. This cable is usually fixed to the bike frame since the brakes of a bike simply try to make the wheels stop, or in the view of a clutch they lock the bike wheel in with the same radial velocity of the frame, which is to say zero. The problem in the flywheel system is that both objects the brake would be trying to connect are rotating, so the cable would get twisted and bound and become unusable.

With these problems on the table, the clutch design required a bit of ingenuity. Rather than having a hydraulic cable to push the caliper onto its new surface, a hollow axle will be used to allow a line to pull the surface onto the caliper. The line will be a brake cable for a simple friction caliper brake. This means that the mechanism for the shifting is an off the shelf part, though the clutch itself will require custom parts. The brake line, when pulled, will pull the large gear into contact with the small gear and thus will shift the gear ratio. This is a much simpler device to construct when compared to a custom continuously variable transmission and most of the parts involved are off the shelf. The drawings for the clutch are shown below.

3.3 Finished design

The two speeds the flywheel will switch between are a 6:1 ratio with the crank and a 2:3 ratio with the crank. Since the crank sprocket has 60 teeth, this means the flywheel will have one 10 tooth sprocket and one 90 tooth sprocket. The 10 tooth sprocket will have six small threaded holes so that it can be fastened directly onto the flywheel's hub. The flywheel will be threaded onto an extended hollow axle that will replace the bike's original back wheel axle. A custom partly threaded axle will attach to the hollow extended axle with a custom connector. The connector will be a hollow cylinder threaded on the inside with enough room to fit the extended hollow axle and the custom partly threaded axle. The custom axle is also hollow and has enough thread to fit inside the connector, but the rest is smooth so that the 90 tooth sprocket can glide freely across it. The custom axle will also have an endcap to prevent the 90 tooth gear from sliding off of the system. The 90 tooth gear will have a large hole on its center to fit the flange. The flange houses both the brake lines end and a bearing. This flange-bearing system will prevent the brake line from taking any of the torque when the clutch is engaged. The flange will have two pieces. One will have an extruding inner cylinder to house the bearing and the brake line's end. It will have one open end. The second piece will cover the open end of the first piece to seal the bearing and brake line in place. Both of the pieces have the same outer diameter and both have threaded holes around their outer diameter that will match threaded holes on the 90 tooth sprocket to fasten all of the parts with this system together. The caliper that will engage or disengage the two gears from each other will have thread holes on it that will allow it to be screwed into the 10 tooth gear. An organized part list with individual pictures is below.



(a) (b)

Figure 6. A picture of the total flywheel system. The flywheel is represented as a simple disc instead of a wheel rim (a) The system in its default state. (b) The charging phase for the flywheel

3.4 Parts list

Extended hollow axle

This is the piece the whole flywheel system rests on. Though there are off-the-shelf hollow bike axles, it is necessary for this part to be custom because the 90 tooth gear needs to be able to slide freely. A connector could be used between the off the shelf part and a custom unthreaded hollow shaft, but this is an unnecessary complication. The outer radius is 9mm and the inner is 7mm, The actual axle will be long enough to be used as the bike's axle as well, but for the purposes of the 3d model it only needs to be long enough for the flywheel system, which is 12cm long.

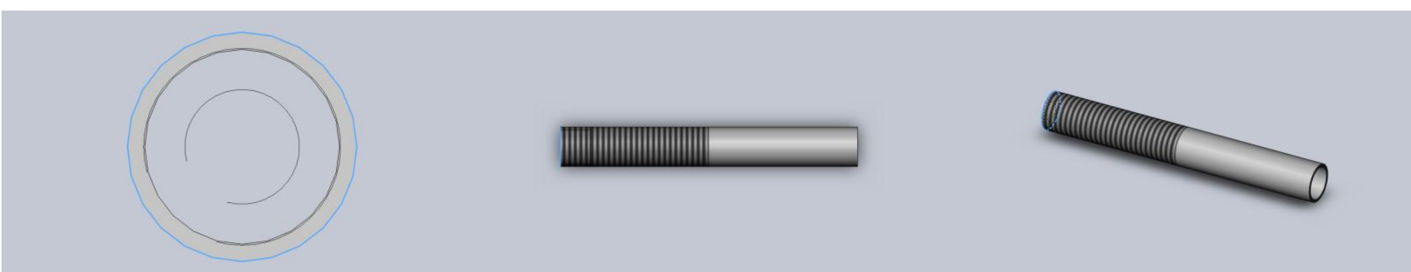


Figure 7. The extended hollow axle from three perspectives

Flange

The flange is where the bearing and the endpiece will be housed. The flange is necessary because it will prevent the line that the endpiece holds from being twisted. The bearing will take the rotational load so that the clutch (the line) can be successfully actuated. The bearing housing will have a diameter of approximately 56mm and a width of approximately 14mm. The total outer diameter will be 100mm.

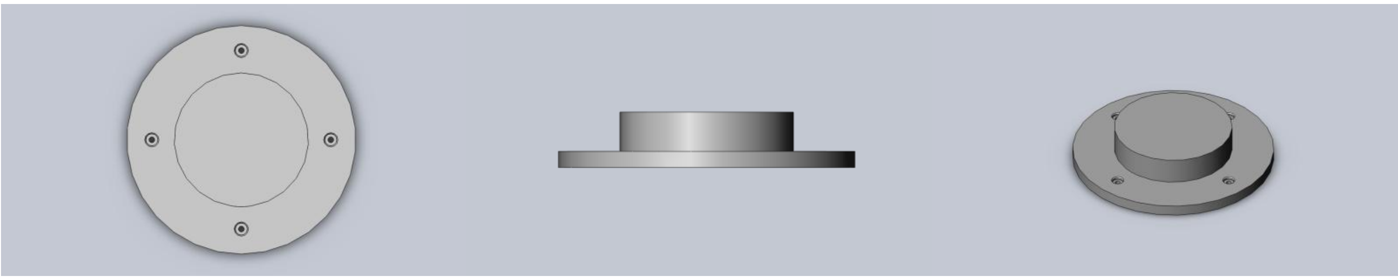


Figure 8. The flange from three perspectives

Flange cover

The flange cover will hold the bearing and the endpiece in place. It has a small extruding piece that will be the part that actually pushes the 90 tooth gear into contact with the caliper. The hole going through the center is 10mm in radius, so it is smaller than the endpiece. This will allow the line to go through while still holding the endpiece in place. The outer radius of the endcap is the same as the flange and has four matching hole to connect the parts.

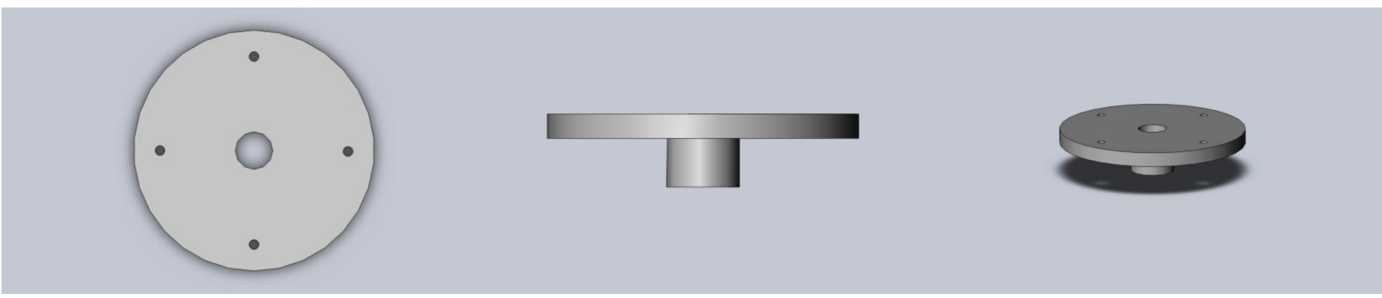


Figure 9. The flange cover from three perspectives

Bearing

The bearing will rest inside the flange and will be covered in oil. The purpose of the bearing is to take the torque load from the flange and not transmit it to the endpiece to allow for a successful clutch actuation. The type of bearing used will be number 6006, which has an outer diameter of 55mm, an inner diameter of 30mm, and a width of 13mm.

Figure 10. A stock picture of a 6006 bearing

Endpiece

The endpiece will hold the line that pulls the flange into contact with the 90 tooth gear. This piece is the most critical in the clutch. The line, which will be the same line used for caliper brake systems, will be tied to the crossbars in the endpiece and then possible glued on for more security. This piece will fit inside the bearing, and will thusly be less than 30mm in diameter and 13mm in width.

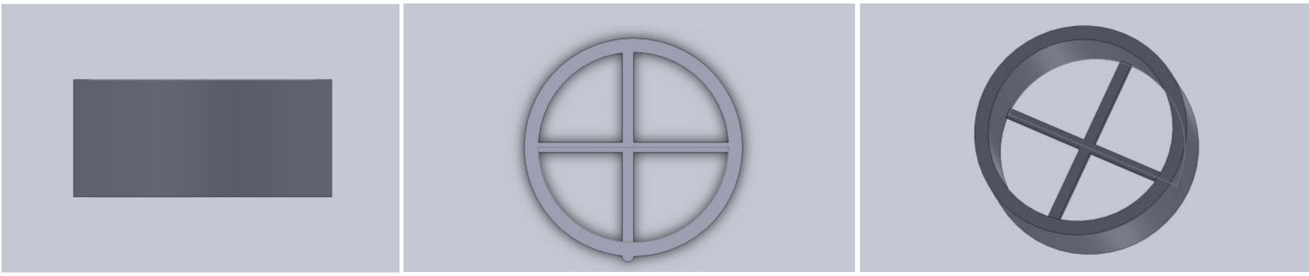


Figure 11. The endpiece from three perspectives

90 tooth gear

The 90 tooth gear will rest on the smooth part of the hollow axle and will either engage with the caliper during the default gear ratio, or disengage with the caliper during the charging phase. This is an off the shelf part, though some modifications may need to be made to lower its weight.

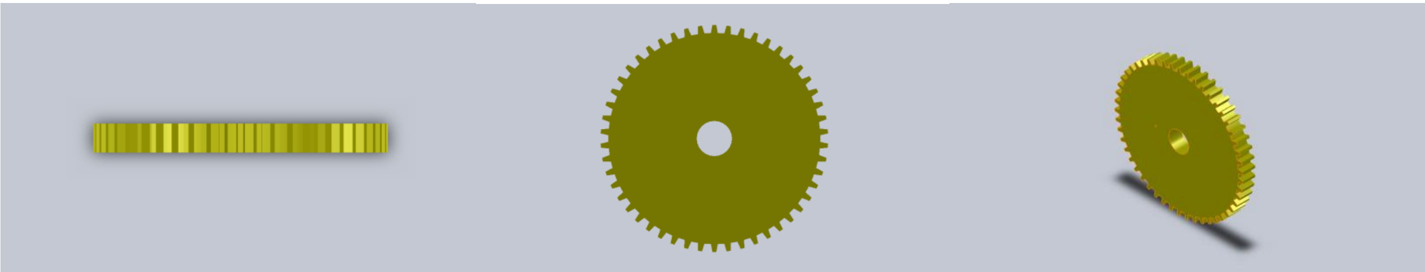


Figure 12. The flange from three perspectives

10 tooth sprocket

The sprocket will connect the flywheel system to the rest of the bike. A chain will connect it to the crank. The sprocket will also have a caliper attached to it to engage the 90 tooth gear when in the default gear ratio. This is an off the shelf part, but it will need to holes bored in it to attach the caliper

Figure 13. The 10 tooth sprocket from three perspectives

Caliper

The caliper is attached to the 10 tooth sprocket and is used as the physical mechanism to switch gear ratios. It will either be in contact with the 90 tooth gear during the default gear ratio or not be in contact with anything during the charging phase. By contacting the 90 tooth gear, the caliper will lock the sprocket into the same rotational speed as the much larger 90 tooth gear which effectively switches the gear ratio.

Figure 14. The caliper from three perspectives

Flywheel (wheel rim)

The flywheel is where the energy is stored from the brake regeneration. A 20 inch diameter bike wheel rim will be used as the flywheel since it will be smaller than the 26 inch wheels on the bike, is an off the shelf part that is easy to get, has most of its mass on the outer edge, would be easy to add weight to if necessary, and has a hub that will be easy to attach to the axle.

Figure 15. A stock picture of a 20 inch bike wheel rim

3.5 Gear shift Mechanism

The actual mechanism to shift the gear ratios in the flywheel system will be the same system used in caliper brakes. This is an off-the-shelf, cheap part that is easy to implement on a bike. The only change will be removing the line from the brake calipers and attaching it to the endpiece. There are many other possible gear shifters that would work, but since this part can easily be purchased and implemented, it is the best choice.

4 Analysis

4.1 Expected Energy Storage without Friction or Air Resistance

First, the problem of how much energy will be stored in the flywheel will be solved without the complication of friction between the road and the bike. This means the angular momentum of the flywheel, the crank, and the two bike wheels will be analyzed for the two different gear ratios. The initial gear ratio between the flywheel and the crank is 3:2, so the relative angular speed of the flywheel will be two thirds of the crank. Since the angular speeds of the two parts are fixed relative to each other, the ratio of their angular momentum can be written as

$$\frac{L_c}{L_{fly}} = \frac{\omega_c}{\omega_{fly}}\frac{I_c}{I_{fly}} = \frac{3}{2}\frac{m_c}{m_{fly}}\frac{r_c^2}{r_{fly}^2} \tag{38}$$

where L is the angular momentum, I is the moment of inertia, m is mass, and r is radius. The subscript c will refer to the crank and the subscript fly will refer to the flywheel. With this ratio, we can put the angular momentum of the crank in terms of the angular momentum of the flywheel.

$$L_c = \frac{3}{2}\frac{m_c}{m_{fly}}\frac{r_c^2}{r_{fly}^2} L_{fly} \tag{39}$$

The same analysis can be done between the crank and the wheel, since they are locked in at a 1:4 ratio for a single speed bike. The angular momentum of the bike wheel in terms of the crank is

$$L_w = 4\frac{m_w}{m_c}\frac{r_w^2}{r_c^2} L_c \tag{40}$$

where the subscript w refers to wheel. Using equation 39, the bike wheels angular momentum in terms of the flywheel's angular momentum is

$$L_w = 6\frac{m_w}{m_{fly}}\frac{r_w^2}{r_{fly}^2}L_{fly} \tag{41}$$

So the angular momentum of the bike in its default gear ratio is

$$L_{initial} = L_{fly} + L_C + 2L_w \tag{42}$$

$$L_{initial} = (1 + \frac{3}{2}\frac{m_c}{m_{fly}}\frac{r_c^2}{r_{fly}^r} + 12\frac{m_w}{m_{fly}}\frac{r_w^2}{r_{fly}^2})L_{fly} \tag{43}$$

$$L_{initial} = \left(m_{fly}r_{fly}^2 + \frac{3}{2}m_c r_c^2 + 12m_w r_w^2\right)c\omega_{fly} \tag{44}$$

where c is a constant determined from the mass distribution.

The second gear ratio between the crank and the flywheel is 1:6. Changing equation 29 based on this new relative speed we get

$$L_C = \frac{1}{6}\frac{m_c}{m_{fly}}\frac{r_c^2}{r_{fly}^2}L_{fly} \tag{45}$$

Equation 40 will remain the same since the gear ratio between the crank and the bike wheel is not changing.

By conservation of momentum,

$$L_{initial} = L_{final} \tag{46}$$

so when switching between the default gear ratio and the charging gear ratio,

$$\left(m_{fly}r_{fly}^2 + \frac{3}{2}m_c r_c^2 + 12m_w r_w^2\right)c\omega_{fly_{initial}} = \left(m_{fly}r_{fly}^2 + \frac{1}{6}m_c r_c^2 + \frac{4}{3}m_w r_w^2\right)c\omega_{fly_{final}} \tag{47}$$

$$\frac{m_{fly}r_{fly}^2 + \frac{3}{2}m_c r_c^2 + 12m_w r_w^2}{m_{fly}r_{fly}^2 + \frac{1}{6}m_c r_c^2 + \frac{4}{3}m_w r_w^2} = \frac{\omega_{fly_{final}}}{\omega_{fly_{initial}}} \tag{48}$$

This equation expresses the percent increase in the flywheel's angular moment solely in terms of easily measured variables.

The energy stored in the flywheel after the charging phase is expressed as

$$E_{final} = \frac{1}{2}I_{fly}\omega_{fly_{final}}^2 \tag{49}$$

substituting equation 48 into equation 49,

$$E_{final} = \frac{1}{2} c m_{fly} r_{fly}^2 \left(\frac{m_{fly} r_{fly}^2 + \frac{3}{2} m_c r_c^2 + 12 m_w r_w^2}{m_{fly} r_{fly}^2 + \frac{1}{6} m_c r_c^2 + \frac{4}{3} m_w r_w^2} \omega_{fly_{inital}}\right)^2 \quad (50)$$

Based on the gear ratios between the flywheel and the wheel, the flywheel's initial speed can be expressed as

$$\omega_{fly_{initial}} = \frac{3}{8} \omega_{w_{initial}} \quad (51)$$

This expression comes from the two gear ratios between the flywheel, the crank, and the wheel. You can also convert rad/s into rpm by the following

$$\omega'_{rpm} = \frac{30}{\pi} \omega_{rad/s} \quad (52)$$

From here on whenever ω is in terms of rpm, it will be expressed as ω' Using equations 41 and 42, equation 40 becomes

$$E_{final} = \frac{\pi^2}{12800} c m_{fly} r_{fly}^2 \left(\frac{m_{fly} r_{fly}^2 + \frac{3}{2} m_c r_c^2 + 12 m_w r_w^2}{m_{fly} r_{fly}^2 + \frac{1}{6} m_c r_c^2 + \frac{4}{3} m_w r_w^2} \omega'_{w_{inital}}\right)^2 \quad (53)$$

Now the energy stored in the flywheel after it is charged is expressed as multiple variables that will be known prior to assembly, and the rpm of the bike, which can be easily measured. This expression does not however take into account the efficiency of the transmission nor rotational friction between the flywheel and the axle.

Making an estimate of the masses and radii involved, we can get an approximate quantitative value for the total amount of energy stored. The mass of the flywheel is 3kg, the mass of the crank is 1 kg, and the mass of the wheel is 4 kilograms. The radius of the flywheel is 25cm, the radius of the crank is 12cm, and the radius of the wheel is 30cm. The constant c is approximately 1 for the flywheel, though this is unrealistic. When getting the bikes wheel initially up to 200RPM, the flywheel will store approximately 100 Joules.

4.2 Flywheel Storage with Air Resistance

If air resistance is taken into account, there will be negligible affects on the flywheel itself, but there will be a significant force applied to the bike and the rider. The bike will slow down during the charging phase and thus the flywheel will store less energy. Therefore the first step is to see how much the bike slows just by switching to the charging phase. If we repeat the process done earlier to relate the final and initial angular velocities of the flywheel by using angular momentum, but instead convert all of the angular velocities into terms of the wheel's angular velocity, then we will have a relation for how much the bike wheels slow down. In terms of the wheel's angular velocity, the relative angular speeds of the rotating parts, based on the different gear ratios, initially are

$$6\omega_{fly_{intial}} = 4\omega_{c_{intial}} = \omega_{w_{intial}} \quad (54)$$

And in the charging phase they are

$$\frac{4}{9}\omega_{fly_{final}} = 4\omega_{c_{final}} = \omega_{w_{final}} \tag{55}$$

Using these relations to rewrite equation 47 in terms of the angular velocity of the wheel, we get

$$\left(\frac{1}{6}m_{fly}r_{fly}^2 + \frac{1}{4}m_c r_c^2 + 2m_w r_w^2\right) c\omega_{w_{initial}} = \left(\frac{9}{4}m_{fly}r_{fly}^2 + 4m_c r_c^2 + 2m_w r_w^2\right) c\omega_{w_{final}} \tag{56}$$

$$\frac{\left(\frac{1}{6}m_{fly}r_{fly}^2 + \frac{1}{4}m_c r_c^2 + 2m_w r_w^2\right)}{\left(\frac{9}{4}m_{fly}r_{fly}^2 + 4m_c r_c^2 + 2m_w r_w^2\right)} = \frac{\omega_{w_{final}}}{\omega_{w_{initial}}} = \frac{\omega'_{w_{final}}}{\omega'_{w_{initial}}} \tag{57}$$

Thus relating the amount of rotational speed lost in the bike wheels due to the change of gear ratio. To convert the rpm of the wheel into the m/s of the bike, the following relation can be used

$$\frac{2\pi r\ \omega'}{60} = v \tag{59}$$

With this relation the bike’s speed can be obtained, and using equation 36, the loss in speed for a given amount of time can be found. Once the amount of speed lost to air resistance is found, using the speed relations in equation 55, the angular velocity of the flywheel can be found, and the energy loss in the flywheel due to air resistance can be found. Using the example from the last section where the bike’s wheels are spinning initially at 200 rpm, after switching to the charging phase the wheels would spin at approximately 140 rpm using the same masses and radiuses as stated earlier. After coasting for five seconds, the rpm in the wheels would go down to 121 rpm, which means the flywheel would store 76 Joules, approximately a 25% loss due to air resistance. At higher speeds air resistance will take an even larger portion of the energy stored in the flywheel.

4.3 Overall Efficiency Gain

Using equation 6 we can find the overall efficiency gain the flywheel system adds. This equation takes the energy of the flywheel as a percent of the total energy of the bike and the rider and subtracts the percent increase in mass the flywheel system adds. Since the angular speed of the flywheel and the total velocity of the bike are related, equation 6 can be simplified to easily measured variables. From equations 55 the relation between the flywheel’s angular speed and the wheel’s angular speed, and to convert angular speed into m/s, the following relation is used

$$\omega = \frac{1}{r}v \tag{60}$$

where r is the radius. Combining equation 55 and 60 for the case of the flywheel we get

$$\omega_{fly} = 7.5v \tag{61}$$

This equation is for when the radius of the wheel is .3m and the bike is in the charging phase. The bike is taken to be in the charging phase because when measuring the efficiency, the flywheel should be about to transfer its energy back into the bike so that the exact amount of energy transferred into the bike and the amount of energy the bike had before that transfer is known. This means that the bike would still be in the charging phase. Using equation 61, equation 6 becomes

$$\varepsilon_{total} = \frac{56.25 m_{fly} r_{fly}^2}{m_{total}} - \frac{m_{fly\ system}}{m_{total}} \tag{62}$$

where η, the efficiency of the transmission has been removed due to the inability to know its value and where c has once again been approximated as one. Also, $m_{fly\ system}$ is the total mass the flywheel system adds to the bike, not just the flywheel itself. Taking the mass of the system as 80kg, the flywheel system as 7kg, and using the above assumptions for the flywheel weight and radius, the result is that the flywheel system adds 4.4% efficiency to the bike. This will be lower in practice due to an imperfect transmission. Also, this only applies when our assumptions about friction being outweighed by air resistance. But, for decently high speeds this flywheel system would increase the efficiency of a bike by a not negligible amount.

4.4 Stress Analysis

The only static stress applied to the flywheel system is the torque applied from the flywheel and the gears. This will create a bending moment on the axle. The approximate weight of the flywheel with the 10 tooth sprocket and caliper is 3 kg, and the 90 tooth gear with the flange is approximately 2kg. Though these weights are small, the axle is thin and hollow, which means that bending stresses are much more significant. The flywheel will be approximately 3.5 cm along the axle and the 90 tooth gear will be 10 cm along the axle. Figure 16(a) shows the free body diagram of the axle, the two negative forces being the flywheel and the gear, and the positive force is the restoring force from the axle's attachment to the bike. A force versus distance graph is shown in b. As shown in equation 26, the area under the curve of this graph represents the bending moment. A graph of the bending moment is shown in c. The peak for the bending moment is 2.05 N*m. Plugging in this bending moment into equation 29, as well as the inner radius of 7mm and the outer radius of 9mm, the bending stress is found to be .87 ksi. This means that there is a double digit safety factor, so the axle should be far sturdy enough to hold the flywheel and the gear.

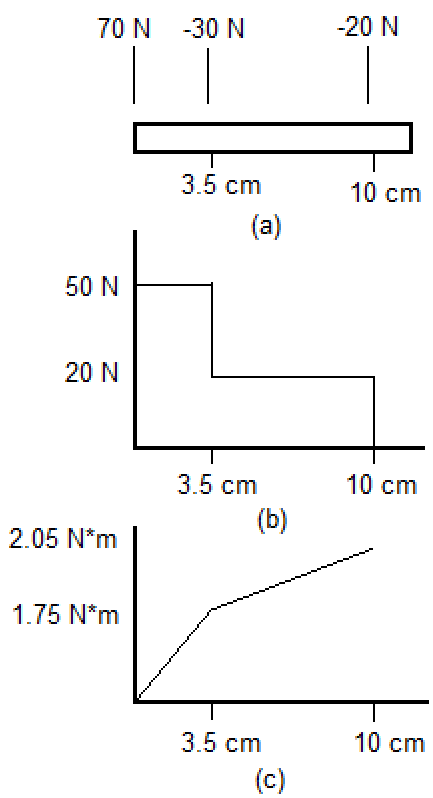


Figure 16. (a) A free body diagram of the system. (b) A force vs. distance graph. (c) A graph of the bending moment vs. distance

5 Conclusion

The design displayed above is a fairly simple implementation of a kinetic energy recovery system with a non-negligible increase in the efficiency of a bicycle. Though there are quite a few parts, many of them are off the shelf and none of the custom parts require highly developed shop skills, such as computer programmed cuts. Also, the installation of the system would not be very complicated outside of actuating the gear shift mechanism, which would mostly involve measuring the correct distance for the line along with some trial and error. Compared to Mr. von Stein's design, it is significantly simpler and lighter. However, the use of a continuously variable transmission allows for the flywheel to spin much faster and is more elegant in its gear shifting mechanism and long term use. The design shown above makes up for its lack of an elegantly design clutch by placing the flywheel in a more optimal spot on the bike and by weighing less. It is impossible for me to use the same measurement of efficiency increase for Mr. von Stein's design because I do not know the rotational speed of the flywheel in his design and I don't know the total weight added to the bike by the flywheel system. But, by his own estimation his kinetic energy recovery system adds ten percent efficiency to the system, and with a fifteen pound flywheel on top of the additional crank, chains, and transmission, it is reasonable to assume that the flywheel system will add approximately ten percent of weight to the bike and its rider, thus canceling out the increased efficiency from stored braking energy. This means despite the design above being simple and unclean in some ways, it promises to be more efficient overall than Mr. von Stein's design. This, however, is contingent on the measured efficiency of the transmission.

Modifications can be made to the design above to make it hypothetically more efficient. Though it would hurt the simplicity of the design, the use of a continuously variable transmission would make the flywheel system better by almost every measure. The flywheel would spin faster and the gear shift mechanism would be more standard. It would be difficult to find a place for the transmission, but this could be overcome by modifying the frame of the bike. Again, this is difficult to implement but is an improvement in the design. Also, the flywheel itself could be heavier to store more energy. This would mean it is harder to accelerate initially, but would give greater boosts to the rider during the trip. When more is known about the measured output of the flywheel, including its energy stored and the efficiency of the transmission, an optimal weight could be selected for maximum efficiency. But this doesn't mean that a person wouldn't want to have additional weight on the flywheel to make it less efficient but more fit to the specific rider's desires.

Flywheel technology is on the rise across many kinds of technology and rightly so. It is a pollution free method of storing energy that has many current and potential applications. In the case of road vehicles there is much to be desired in terms of energy efficiency, especially when considering pollution per unit of energy. Any system of brake regeneration can help that, but flywheels have the potential to increase the efficiency of road vehicles without direct or indirect negative effects on the environment. The batteries used in hybrids do not last the cars lifetime and can have costly environmental effects. A flywheel has environmental impact only at its time of production, and has the potential to heavily outweigh those costs through its use. Bikes do not have the pollution problems cars and other modes of transportation have, but they can serve as a good analogy for how a kinetic energy recovery system can increase the efficiency of a vehicle.

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